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Essays in Business Cycles and Asset Pricing

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ESSAYS IN BUSINESS CYCLES AND ASSET PRICING

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ESSAYS IN BUSINESS CYCLES AND ASSET PRICING

A Dissertation Presented to the Graduate Faculty of the

Dedman College

Southern Methodist University

in

Partial Fulfillment of the Requirements

for the degree of

Doctor of Philosophy

with a

Major in Economics

by

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This dissertation investigates several key macroeconomic and asset pricing topics, with a particular interest in exploring the underlying driving forces of business cycles and asset market fluctuations. The dissertation includes three chapters, with the first two chapters solo-authored by me and the third chapter co-authored with Dr. Carlos Zarazaga.

The first chapter develops a dynamic and stochastic general equilibrium model, and exploits Bayesian inference methods to investigate the major sources of fluctuations in aggregate variables and asset prices. Taking into account the possibility that the growth of total factor productivity, labor-augmenting technology and investment-specific technology might consist of permanent and transitory components, I consider a baseline along with three alternative specifications on the structure of the exogenous processes. It is found that the identification of the major sources of aggregate fluctuations hinges critically upon researchers' assumptions on the exogenous processes. Bayesian model comparison indicates that previous studies might have overlooked the persistent change in investment-specific technology growth, and thus, underestimate its importance to driving the business cycles. Using the structure of the exogenous processes that is mostly favored by the data, I find that investment-specific technology shocks contribute a significantly large fraction of the short-run and the long-run fluctuations in output growth, investment growth, and the share of total market values in output. In addition, the long-run predicted error variance of consumption-output ratio and hours is overwhelmingly due to shocks to the permanent and the transitory

components of investment-specific technology. In contrast, labor augmenting technology shocks, preference shocks and government spending shocks are only important contributors to fluctuations in hours, consumption and government expenditures, respectively, in the very short-run.

The second chapter proposes a macro-based asset pricing model, and seeks to identify the macroeconomic driving forces of asset price movements. The long-run risks literature highlights the importance of the predictable long-run component in consumption growth to explaining the asset pricing facts, but might overlook other potential determinants that are not consumption-related. So as to understand the asset market phenomena from a wider perspective, I develop a consumption-based asset pricing model with recursive preferences, accommodating both the long-run consumption growth and the time-preference shock channels. In the modeled economy, asset market fluctuates in response to long-run consumption growth, time-preference shocks and their respective conditional volatilities; and the expected equity premium reflects the market compensation for households' exposure to consumption growth uncertainty and valuation risks. Empirical evidence, based on the moment-matching methods and the particle smoothing algorithm, indicates that, first, the proposed model is able to replicate the joint dynamics of the key asset market variables. Second, compared with the standard long-run risks model, the proposed framework achieves remarkable improvement along the dimension of resolving the major asset pricing puzzles. In addition, it is found that long-run consumption growth is the major contributor to asset market fluctuations, whereas time-preference shocks and valuation risks are non-negligible determinants of the risk-free rate.

The third chapter develops a novel methodology for systematic assessment of the credibility of fiscal stabilization programs. The credibility of fiscal stabilization programs plays a critical role in their macroeconomic outcomes, yet formal assessments of that credibility are typically missing from analyses of the economic consequences and effectiveness of those programs. Therefore, we remedy that omission for the most recent consolidation attempt in the U.S.: the 2011-mandated budget sequestration spending cuts in discretionary spending

slated to begin in 2013. The proposed methodology draws its elements from the “event-study” and the Business Cycle Accounting traditions. It is found that the fiscal austerity program had little, if any, credibility during the relevant 2012 - 2013 event-study window. A major implication of our findings is that the policy recommendations suggested by the observed outcomes of fiscal stabilization programs might be misleading, absent consideration of the extent to which they were perceived as sustainable.

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To Jinghua, Shuqin, Xuchang and My Family

CHAPTER 1

STRUCTURE OF EXOGENOUS PROCESSES, AGGREGATE FLUCTUATIONS, AND ASSET PRICE MOVEMENTS: BAYESIAN ESTIMATION OF A DSGE MODEL

1.1. Introduction

Over the past few decades, a large body of the real business cycle literature has attempted to identify the major sources of aggregate fluctuations. Exploiting the Dynamic and Stochastic General Equilibrium (DSGE) framework and taking into account various aspects of the real rigidities in the economy, recent business cycle studies have achieved remarkable improvements along the dimension of reproducing the key stylized facts of the post war U.S. data in the simulated environment. However, the consensus of the key driving forces of the business cycles has been barely reached among economists, and the debate remains.

Before late 1980s, the conventional wisdom of the business cycle literature suggests that fluctuations in aggregate variables are overwhelmingly due to macroeconomic innovations to total factor productivity and labor-augmenting technology. Greenwood, Hercowitz and Huffman (1988), however, highlight the importance of investment-specific technology shocks to driving the business cycles. The view that investment-specific technology shocks play a non-negligible role in accounting for the business cycle facts is further supported by Greenwood, Hercowitz and Krusell (1997) and Fisher (2006), but challenged later by Justiniano, Primiceri, and Tambalotti (2011). In particular, Justiniano, Primiceri, and Tambalotti (2011) consider two investment innovations that potentially affect capital accumulation at the aggregate level. Those are, the standard investment-specific technology shocks that govern the efficiency in transforming consumption goods into investment goods, and the shocks to the marginal efficiency of investment which regulate the transformation rate of investment goods into capital. According to their empirical findings, the second shocks explain over 50% of the

predicted error variance of output, hours and investment, whereas the standard investment-specific technology shocks play no role in driving the business cycles. Along the line of effort that disentangles the role of investment-related innovations, Schmitt-Grohé and Uribe (2012) take into account news (or anticipated) shocks under a medium-scaled DSGE framework where households have Jaimovich-Rebelo preferences, and provide empirical evidence largely consistent with that of Justiniano, Primiceri, and Tambalotti (2011).

So as to fully understand the causes and the consequences of the business cycles, it seems natural to carefully investigate the potential sources leading to the aforementioned empirical discrepancies. Inarguably, inconsistencies in model implications have to be accounted for by varying specifications on the underlying analytical framework and the identification procedures adopted for the quantitative practice. Existing studies have unambiguously demonstrated that the identification of the major contributors to business fluctuations is largely affected by the data incorporated in the observable vector. For instance, Avdjiev (2009) and Schmitt-Grohé and Uribe (2012) show that estimation of DSGE models with and without asset market data can yield remarkably distinct model-implied aggregate dynamics. In particular, Schmitt-Grohé and Uribe (2012) report that investment-specific technology shocks play trivial role when the data on the relative price of investment is incorporated into the observable vector, but become a significant contributor to aggregate fluctuations once excluded.

For the extent to which the identification of the major sources of business fluctuations is contingent upon the analytical framework adopted for the quantitative analysis, the existing literature provides quite limited guidance. The rarity of work along that line of effort is not necessarily incomprehensible. First, each study investigates a uniquely specific topic, and has its own focuses and concerns. Therefore, the assumption made on the modeled economy is completely up to the underlying research purposes, and hence, needs to be respected. In addition, comparing empirical evidence across analytical frameworks with fundamentally distinct features seems not a feasible research avenue for reconciling the inconsistencies between previous studies, because the impact of fundamental differences in modeling choices (such

as preference specifications, incorporation and exclusion of certain sectors, and so forth) on identifying the business cycle drivers seems almost not quantitatively traceable.

Given these concerns, this study investigates the major business cycle contributors and attempts to shed light on the potential sources of the documented empirical discrepancies from a novel perspective. To be specific, conditional on a proposed economy capable of replicating the key features of its actual counterpart, this study constructs a set of competing alternatives through varying exclusively the assumptions on the data generating processes of the exogenous variables, and seeks to assess whether the model implications on the key business cycle drivers are sensitive to these small changes in model specifications when the remaining features of the modeled economy are well retained. We restrict our attention to the structure of stochastic processes taken by the exogenous variables not only due to its apparent tractability, but also because its importance to understanding the business cycle phenomena has been constantly overlooked. Previous studies in the business cycle literature usually assume that the processes of the exogenously determined variables take stationary auto-regressive forms. This conventional specification, however, normally does a poor job in capturing the behavior of inpersistent variables whose long-run movements are otherwise persistent. Given that the true data generating processes of the exogenous variables are unobservable, precluding the possible existence of persistent long-run components embedded can potentially lead to remarkably distinct, if not jeopardized, model implications. For instance, Bansal and Yaron (2004) demonstrate that, under their partial equilibrium framework where households have Epstein-Zin recursive preferences, several key asset pricing puzzles can be largely resolved when consumption growth is specified as a white noise process consisting of a small long-run component.

In this paper, we develop a neoclassical growth model under which aggregate dynamics and asset price movements are driven by innovations to technology growth, preferences and government expenditures. Our theoretical framework is closely related to those of Schmitt-Grohé and Uribe (2008, 2012), and Avdjiev (2009). The proposed economy is augmented with four real rigidities, namely variable capacity utilization, capital adjustment costs, and

internal habit formation in consumption and leisure. For each model specification, we exploit Bayesian techniques to estimate the unknown economic parameters, and then perform variance decomposition and impulse responses analysis to draw model inference. We find that the identification of the business cycle drivers hinges critically on the structure of the exogenous processes. Under the specifications where investment-specific technology growth consists of a permanent component, a vast majority of the fluctuations in output growth, investment growth, hours and total market values is accounted for by shocks to investment-specific technology growth. In contrast, when a permanent component is embedded in labor-augmenting technology growth but not investment-specific technology growth, the variance decomposition statistics indicate that the major contributors to business cycles are shocks to total factor productivity and labor-augmenting technology growth. While consistent with the findings of Schmitt-Grohé and Uribe (2012), this specification, however, is not supported by the U.S. data. Bayesian odd ratio test indicates that previous studies might have overlooked the persistence in investment-specific technology growth, and hence, underestimate its importance to explaining the business cycle facts.

The rest of this paper is organized as follows. We introduce our theoretical framework in section 1.2. Section 1.3 describes the baseline and the three alternative specifications on the structure of the exogenous processes. Estimation procedure, calibration choices and parameter estimates are presented in section 1.4. Model inference based on variance decomposition and Bayesian model comparison is discussed in section 1.5. Section 1.6 reports the impulse responses analysis. And section 1.7 concludes.

1.2. The Model

In this study, we consider an economy with a stand-in representative household, whose preference over consumption and leisure is given by

$$E_t \sum_{i=0}^{\infty} \beta^{t+i} e^{\xi_{t+i}} \left\{ \frac{[(C_{t+i} - b_c C_{t-1+i})(L_{t+i} - b_l L_{t-1+i})^\chi]^{1-\gamma} - 1}{1-\gamma} \right\},$$

where $\beta \in (0, 1)$ denotes household's subjective discount factor; ξ_t represents the stochastic preference shock at time t ; C_t and L_t denote the consumption goods and leisure consumed by the household, respectively; $b_c \in [0, 1]$ and $b_l \in [0, 1]$ capture the degree of internal habit formation in consumption and leisure, respectively; χ governs the Frisch elasticity of labor supply; and γ denotes the inverse of intertemporal elasticity of substitution (IES).

Let H_t denote hours worked by the representative household. Normalizing total hours to unity yields $L_t + H_t = 1$ for all t . Rewriting the life-time utility function in C_t and H_t , we have

$$E_t \sum_{i=0}^{\infty} \beta^{t+i} e^{\xi_{t+i}} \left\{ \frac{[(C_{t+i} - b_c C_{t-1+i}) (\tau + b_l H_{t-1+i} - H_{t+i})^\chi]^{1-\gamma} - 1}{1 - \gamma} \right\}, \quad (1.1)$$

where $\tau = 1 - b_l$ is constant.

Assume that the representative household is the owner of physical capital. The law of motion of capital is specified as follows,

$$K_{t+1} = [1 - \delta(u_t)] K_t + e^{z_t^I} I_t \left[1 - \Phi \left(\frac{I_t}{I_{t-1}} \right) \right], \quad (1.2)$$

where K_t , predetermined in period $t - 1$, represents the capital stock in period t ; I_t denotes the level of gross investment; z_t^I is interpreted as a transitory investment-specific productivity disturbance that governs the efficiency in transforming investment goods into physical capital; and u_t denotes the time-varying rate of capital utilization. By definition, the effective amount of capital available for firms' production in each period is given by $u_t K_t$. We assume that the depreciation rate of capital $\delta(\cdot)$ is an increasing and convex function of u_t , which is given by

$$\delta(u_t) = \delta_0 + \delta_1 (u_t - 1) + \frac{\delta_2}{2} (u_t - 1)^2. \quad (1.3)$$

In equation (1.3), δ_0 refers to the non-stochastic steady-state depreciation rate; δ_1 is determined by the steady-state equilibrium conditions when u_t is normalized to unity; and δ_2 regulates the sensitivity of capital utilization to variation in the rental rate of capital.¹

¹For details regarding how δ_1 is determined, please refer to Appendix A.2.

In equation (1.2), the function $\Phi(\cdot)$ captures the investment adjustment cost, which is specified as follows,

$$\Phi\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - \mu^i\right)^2, \quad (1.4)$$

where κ is constant; and μ^i is the steady state growth rate of investment.

In addition, we assume that the representative household's sequential budget constraint is given by

$$C_t + A_t I_t + \Gamma_t = W_t H_t + r_t u_t K_t + P_t. \quad (1.5)$$

In equation (1.5), A_t denotes the non-stationary stochastic productivity factor which affects the technology rate of transforming consumption goods into investment goods; W_t and r_t denote the competitive wage of labor supply and the rental rate of capital, respectively; P_t represents the profit that the household collects from the firm; and Γ_t is the lump-sum tax paid to the government.² Given A_t 's non-stationarity, we define the gross growth rate of A_t as

$$\mu_t^a = \frac{A_t}{A_{t-1}}, \quad (1.6)$$

and assume that μ_t^a follows a stationary stochastic process with steady-state value equal to μ^a .

According to the above model specification, the representative household's optimization problem is choosing a sequence of $\{C_t, H_t, I_t, K_{t+1}, u_t\}_{t=0}^{\infty}$ to maximize the objective function (1.1), subject to equations (1.2) and (1.5), and taking as given the stochastic processes $\{\xi_t, Z_t^I, A_t, W_t, P_t\}_{t=0}^{\infty}$ and the initial condition C_{-1}, I_{-1} and K_0 .

For the production side of the modeled economy, we assume that the representative firm uses effective capital stock and labor as inputs, and its production function takes the following Cobb-Douglas form:

$$Y_t = e^{z_t} F(u_t K_t, X_t H_t) = e^{z_t} (u_t K_t)^\alpha (X_t H_t)^{1-\alpha}, \quad (1.7)$$

²We assume that the representative household owns the firm.

where Y_t denotes output; z_t is total factor productivity shock; and X_t is a non-stationary labor-augmented technology shock. We further define the gross growth rate of X_t as

$$\mu_t^x = \frac{X_t}{X_{t-1}}. \quad (1.8)$$

We assume that μ_t^x follows a stationary process with steady-state value μ^x .

We assume that the government in each period consumes an exogenous amount of resource G_t , which is financed by levying lump-sum taxes. Inspired by Schmitt-Grohé and Uribe (2012), we assume that G_t has a stochastic trend component, X_t^G , which is specified as follows,

$$X_t^G = (X_{t-1}^G)^{\rho_{xg}} (X_{t-1}^Y)^{1-\rho_{xg}}, \quad (1.9)$$

where ρ_{xg} is a constant. According to our specification, first, government expenditures and output are allowed to be cointegrated, which ensures that the share of government expenditures in output is stationary. Second, the trend of government consumption X_t^G is potentially smoother than the trend of output, namely X_t^Y , and the degree of smoothness is regulated by ρ_{xg} . In addition, given that X_t^G is determined by the information available in period $t-1$, equation (1.9) permits lagged responses of X_t^G to contemporaneous changes in the trend component of aggregate output. These features are consistent with the stylized facts of the post war U.S. data.

It is straightforward to show that the aggregate resource constraint takes the following form:

$$C_t + A_t I_t + G_t = Y_t. \quad (1.10)$$

Given that the modeled economy is free of distortions, solving the competitive equilibrium allocation is equivalent to solving a social planner's problem. Let A_t and $A_t Q_t$ denote the Lagrangian multipliers on equations (1.10) and (1.2). Then, the corresponding social planner's

problem can be formulated as

$$\begin{aligned}
& \max_{C_{t+i}, H_{t+i}, I_{t+i}, K_{t+1+i}, u_{t+i}} E_t \sum_{i=0}^{\infty} \beta^{t+i} \left\{ e^{\xi_{t+i}} \frac{[(C_{t+i} - b_c C_{t-1+i})(\tau + b_l H_{t-1+i} - H_{t+i})^\chi]^{1-\gamma} - 1}{1-\gamma} \right. \\
& + A_{t+i} [e^{z_{t+i}} (u_{t+i} K_{t+i})^\alpha (X_{t+i} H_{t+i})^{1-\alpha} - (C_{t+i} + A_{t+i} I_{t+i} + G_{t+i})] \\
& \left. + A_{t+i} Q_{t+i} \left[[1 - \delta(u_{t+i})] K_{t+i} + e^{z_{t+i}^I} I_{t+i} \left[1 - \Phi\left(\frac{I_{t+i}}{I_{t-1+i}}\right) \right] - K_{t+1+i} \right] \right\}.
\end{aligned}$$

Taking as given the set of exogenous stochastic processes $\{\xi_t, z_t, A_t, X_t, z_t^I, G_t\}_{t=0}^{\infty}$ and the initial values of C_{-1} , I_{-1} and K_0 , we solve the optimization problem faced by the social planner, and obtain a competitive equilibrium where the set of stochastic processes $\{C_t, H_t, I_t, K_{t+1}, u_t, Y_t, \Lambda_t, Q_t\}_{t=0}^{\infty}$ satisfies equations (1.2), (1.7), (1.10), as well as the following first order conditions:

$$\begin{aligned}
\Lambda_t &= e^{\xi_t} (C_t - b_c C_{t-1})^{-\gamma} (\tau + b_l H_{t-1} - H_t)^{\chi(1-\gamma)} \\
&\quad - \beta b_c E_t \left\{ e^{\xi_{t+1}} (C_{t+1} - b_c C_t)^{1-\gamma} (\tau + b_l H_t - H_{t+1})^{\chi(1-\gamma)} \right\};
\end{aligned} \tag{1.11}$$

$$\begin{aligned}
\Lambda_t e^{z_t} (1 - \alpha) (u_t K_t)^\alpha (X_t H_t)^{-\alpha} &= \chi e^{\xi_t} (C_t - b_c C_{t-1})^{-\gamma} (\tau + b_l H_{t-1} - H_t)^{\chi(1-\gamma)-1} \\
&\quad - \chi \beta E_t \left\{ e^{\xi_{t+1}} (C_{t+1} - b_c C_t)^{-\gamma} (\tau + b_l H_t - H_{t+1})^{\chi(1-\gamma)-1} \right\};
\end{aligned} \tag{1.12}$$

$$\begin{aligned}
A_t \Lambda_t &= \Lambda_t Q_t e^{z_t^I} \left[1 - \Phi \left(\frac{I_t}{I_{t-1}} \right) - \left(\frac{I_t}{I_{t-1}} \right) \Phi' \left(\frac{I_t}{I_{t-1}} \right) \right] \\
&+ \beta E_t \left\{ \Lambda_{t+1} Q_{t+1} e^{z_{t+1}^I} \left(\frac{I_{t+1}}{I_t} \right)^2 \Phi' \left(\frac{I_{t+1}}{I_t} \right) \right\};
\end{aligned} \tag{1.13}$$

$$\Lambda_t Q_t = \beta E_t \left\{ \Lambda_{t+1} \left[e^{z_{t+1}} \alpha u_{t+1} (u_{t+1} K_{t+1})^{\alpha-1} (X_{t+1} H_{t+1})^{1-\alpha} + Q_{t+1} (1 - \delta(u_{t+1})) \right] \right\}; \tag{1.14}$$

$$e^{z_t} \alpha u_t (u_t K_t)^{\alpha-1} (X_t H_t)^{1-\alpha} = Q_t \delta'(u_t). \tag{1.15}$$

Notice that Q_t can be interpreted as the marginal Tobin's q , which captures the relative price of capital stock to consumption goods.

Similar to the standard real business cycle models in the literature, the proposed model in this paper can be extended straightforwardly to capture the asset market fluctuations driven by responses of market fundamentals to macroeconomic shocks. First, firms' profit in each period is given by

$$P_t = Y_t - W_t H_t - A_t I_t.$$

Since the equilibrium wage in the competitive labor market is equal to the marginal productivity of labor, we have

$$W_t = (1 - \alpha) e^{z_t} (u_t K_t)^\alpha (X_t H_t)^{-\alpha} = (1 - \alpha) \frac{Y_t}{H_t}.$$

Therefore, firm's profit can be rewritten as

$$P_t = \alpha Y_t - A_t I_t. \tag{1.16}$$

In addition, it can be shown that the one period ahead gross risk-free rate R_t^{rf} satisfies

$$R_t^{rf} = \frac{1}{\beta} E_t \left(\frac{\Lambda_t}{\Lambda_{t+1}} \right). \quad (1.17)$$

Then, the end-of-period firm value, V_t , satisfies the following recursive condition:

$$V_t = \beta E_t \left[\left(\frac{\Lambda_{t+1}}{\Lambda_t} \right) (V_{t+1} + P_{t+1}) \right]. \quad (1.18)$$

Therefore, in the proposed economy which consists of the representative household, the representative firm, the government, and the asset market, the complete set of equilibrium conditions is given by equations (1.2), (1.7), (1.10), and (1.11) - (1.18). Given that these equilibrium conditions are characterized by non-stationary aggregate variables, however, a steady state does not exist. In order to obtain a stationary state-space representation of the equilibrium conditions, we transform these equilibrium conditions into their corresponding stationary form via detrending the non-stationary variables by their respective trends. Detailed discussion is provided in Appendix A.1.

1.3. Structure of Exogenous Processes

Previous studies in the real business cycle literature typically assume that the processes of exogenously determined variables take stationary auto-regressive forms. While straightforward to implement, this conventional specification overlooks the fact that those exogenous variables are usually unobserved, and thus precludes the possibility that some processes might be persistent in nature. Even though estimating the auto-regression coefficients of the unobservables can be informative about their transition dynamics, the auto-regression specification does not suffice to reasonably capture the behavior of inpersistent variables whose long-run movements are otherwise persistent. As suggested in Shephard and Harvey (1990), it is almost impossible to exploit finite samples to distinguish between a pure white noise process and a white noise process with a small persistent component. And neglecting the seemingly tiny difference in data generating process can potentially lead to remarkably

distinct model implications. A well-known example can be borrowed from the long-run risks literature. Unlike the conventional consumption-based asset pricing studies which assume that consumption growth is pure white noise, Bansal and Yaron (2004) propose that there exists a small persistent component embedded in the consumption growth process, and find that the incorporation of long-run consumption growth helps to resolve several major asset pricing puzzles.

Due to the aforementioned concern, we choose to decompose three key exogenous variables, namely total factor productivity growth (z_t), labor-augmenting technology growth (μ_t^x) and investment-specific technology growth (μ_t^a), into permanent and transitory components. By definition, a permanent component of a given process regulates its long-run persistence, and a transitory component governs its transitory dynamics.³ Since one of our major goals is to investigate whether the identification of the major driving forces of aggregate fluctuations is contingent upon researchers' specifications on the structure of the exogenous processes, this study considers one baseline model along with three alternatives. In addition, note that it is not impossible that the structure of the exogenous processes is misspecified if the unobserved true data generating process of a given variable does not contain a permanent component. Therefore, it seems necessary to investigate a set of competing alternatives, which would allow us to perform model comparison (using Bayesian inference techniques) to minimize the impact of misspecification, and eventually identify the specification that is mostly favored by the data.

To be specific, for the growth rate of labor-augmenting technology and investment-specific technology, we define the percentage deviation of μ_t^x and μ_t^a from their respective steady state as

$$\begin{aligned}\widehat{\mu_t^x} &= \log \left(\frac{\mu_t^x}{\mu^x} \right); \\ \widehat{\mu_t^a} &= \log \left(\frac{\mu_t^a}{\mu^a} \right);\end{aligned}$$

³Similar specification can be found in Balke and Wohar (2006a).

or simply,

$$\widehat{\mu}_t^j = \log \left(\frac{\mu_t^j}{\mu^j} \right) \quad \text{for } j = \{x, a\} .$$

In the baseline model, we assume that $\widehat{\mu}_t^j$ consists of a permanent component $\widehat{\mu}_t^{j,P}$, and a transitory component $\widehat{\mu}_t^{j,T}$:

$$\widehat{\mu}_t^j = \widehat{\mu}_t^{j,P} + \widehat{\mu}_t^{j,T} . \quad (1.19)$$

We further specify the processes of these two components as follows,

$$\widehat{\mu}_t^{j,P} = \phi^{j,P} \widehat{\mu}_{t-1}^{j,P} + \epsilon_t^{j,P}; \quad (1.20)$$

$$\widehat{\mu}_t^{j,T} = \phi_1^{j,T} \widehat{\mu}_{t-1}^{j,T} + \phi_2^{j,T} \widehat{\mu}_{t-2}^{j,T} + \epsilon_t^{j,T} . \quad (1.21)$$

First, equation (1.20) captures the persistent long-run movements in $\widehat{\mu}_t^j$. To avoid introducing non-stationarity to the equation system, we assume that $\phi^{j,P}$ is close to, but strictly less than, one. Second, the transitory component $\widehat{\mu}_t^{j,T}$ is assumed to follow an $AR(2)$ process with auto-regression coefficients $\phi_1^{j,T}$ and $\phi_2^{j,T} \in (0, 1)$. In addition, $\epsilon_t^{j,P}$ and $\epsilon_t^{j,T}$ are orthogonal *i.i.d.* innovations to $\widehat{\mu}_t^{j,P}$ and $\widehat{\mu}_t^{j,T}$, respectively. The mean and variance of these shocks are given by

$$\begin{bmatrix} \epsilon_t^{j,P} \\ \epsilon_t^{j,T} \end{bmatrix} \sim N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{j,P}^2 & 0 \\ 0 & \sigma_{j,T}^2 \end{bmatrix} \right) .$$

For total factor productivity growth, it is also assumed that there are permanent and transitory components embedded in the process of z_t :

$$z_t = z_t^P + z_t^T; \quad (1.22)$$

$$z_t^P = \phi^{z,P} z_{t-1}^P + \epsilon_t^{z,P}; \quad (1.23)$$

$$z_t^T = \phi_1^{z,T} z_{t-1}^T + \phi_2^{z,T} z_{t-2}^T + \epsilon_t^{z,T}; \quad (1.24)$$

$$\begin{bmatrix} \epsilon_t^{z,P} \\ \epsilon_t^{z,T} \end{bmatrix} \sim N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{z,P}^2 & 0 \\ 0 & \sigma_{z,T}^2 \end{bmatrix} \right).$$

Once again, $\phi_1^{z,T}$ and $\phi_2^{z,T}$ are auto-regression coefficients that lie within $[0, 1]$; $\phi^{z,P}$ is close to, but strictly less than, one; and $\epsilon_t^{z,P}$ and $\epsilon_t^{z,T}$ are *i.i.d.* normal shocks.

For the rest of the exogenous variables, we simply assume that they follow stationary $AR(2)$ processes with no permanent components embedded:

$$g_t = \phi_1^g g_{t-1} + \phi_2^g g_{t-2} + \epsilon_t^g, \quad (1.25)$$

$$z_t^I = \phi_1^{z^I} z_{t-1}^I + \phi_2^{z^I} z_{t-2}^I + \epsilon_t^{z^I}, \quad (1.26)$$

$$\xi_t = \phi_1^\xi \xi_{t-1} + \phi_2^\xi \xi_{t-2} + \epsilon_t^\xi, \quad (1.27)$$

where $g_t \equiv \log(\widetilde{G}_t/\widetilde{G})$ in equation (1.25) refers to the percentage deviation of the growth rate of government expenditures from its steady state; ϕ_1^s and ϕ_2^s , for $s = \{g, z^I, \xi\}$, denote auto-regression coefficients; ϵ_t^s is *i.i.d.* normal innovations to variable s_t with mean zero and standard deviation σ_s .

Note that the structure of exogenous processes of z_t , g_t , ξ_t and z_t^I stays fixed across all four model specifications. For alternative specifications, the modeling assumption varies exclusively on the permanent components of $\widehat{\mu}_t^x$ and $\widehat{\mu}_t^a$. Under specification 2, we eliminate the permanent component of $\widehat{\mu}_t^x$ from its process, and simply assume that the dynamics of $\widehat{\mu}_t^x$ is only captured by the transitory component. Formally, keeping the rest of the exogenous structure fixed, $\widehat{\mu}_t^x$ takes the following form:

$$\widehat{\mu}_t^x \equiv \widehat{\mu}_t^{x,T} = \phi_1^{x,T} \widehat{\mu}_{t-1}^{x,T} + \phi_2^{x,T} \widehat{\mu}_{t-2}^{x,T} + \epsilon_t^{x,T}. \quad (1.28)$$

Under specification 3, the permanent component of $\widehat{\mu}_t^a$ is excluded in an analogous fashion. And under the last model specification, neither $\widehat{\mu}_t^x$ nor $\widehat{\mu}_t^a$ is assumed to exhibit persistent long-run growth. Modeling assumption for each specification is summarized in Table 1.1.

Table 1.1. Structure of Exogenous Processes: Baseline and Three Alternatives

	z_t		$\widehat{\mu}_t^x$		$\widehat{\mu}_t^a$	
	Permanent	Transitory	Permanent	Transitory	Permanent	Transitory
Baseline	✓	✓	✓	✓	✓	✓
Specification 2	✓	✓	—	✓	✓	✓
Specification 3	✓	✓	✓	✓	—	✓
Specification 4	✓	✓	—	✓	—	✓

Note: “✓” indicates that the variable consists of that component; and “—” indicates that the variable does not contain that component.

1.4. Bayesian Estimation

1.4.1. Data and Estimation Procedure

In this study, we apply Bayesian method to the state-space representation of the linearized equilibrium conditions and estimate the unknown structural parameters for all model specifications. In particular, we exploit the Markov Chain Monte Carlo (MCMC) approach with a Random Walk Metropolis-Hastings (RW-MH) algorithm to improve computational efficiency. Our RW-MH sampling algorithm generates 100,000 draws from the proposed distribution and has a 50,000-draw burn-in period. The procedure of our Bayesian estimation is standard in the literature.

To estimate the unknown deep parameters, we use quarterly U.S. data ranging from 1949:Q1 to 2006:Q4. The vector of observables Ω_t is listed as follows,

$$\Omega_t = \left[\Delta \log(Y_t), \Delta \log(A_t I_t), \log(C_t/Y_t), \log(G_t/Y_t), \log(V_t/Y_t), H_t, R_t^{rf} \right]', \quad (1.29)$$

where $\Delta \log(Y_t)$ refers to the growth rate of real per capita GDP; $\Delta \log(A_t I_t)$ denotes the growth rate of real per capita investment; (C_t/Y_t) , (G_t/Y_t) and (V_t/Y_t) denote the shares of real per capita consumption, real per capita government expenditures and real per capita

total market values in real per capita output, respectively; H_t refers to hours worked; and R_t^{rf} is the real risk-free rate. Detailed data description is provided in Appendix A.3.

1.4.2. Calibration

In this study, we calibrate a small set of the structural parameters that regulate the dynamics of the aggregate variables in the modeled economy. First, as summarized in Table 1.2, the representative household's subjective discount factor β is calibrated at 0.99; the steady state share of capital stock in output α is set at 0.3; the steady state depreciation rate and capital utilization rate are calibrated at 0.025 and 1, respectively; and the steady state government-output ratio is set equal to 0.2. Note that these calibration choices are widely used in the real business cycle literature.

Second, following Schmitt-Grohé and Uribe (2012), we set the steady state growth rates of labor-augmenting technology and investment-specific technology at 1.00165 and 0.9957, respectively. Given the calibrated values of μ^x , μ^a and α , the implied steady state values of μ^y and μ^k are 1.0045 and 1.0033, respectively.

In addition, the steady state values of the exogenous variables (namely $\widehat{\mu_t^{x,P}}$, $\widehat{\mu_t^{x,T}}$, $\widehat{\mu_t^{a,P}}$, $\widehat{\mu_t^{a,T}}$, z_t^P , z_t^T , g_t , z_t^I and ξ_t) are all zero. So as to capture the persistent changes in the permanent components (yet without introducing non-stationarity to the equation system), the auto-regression coefficients of $\phi^{x,P}$, $\phi^{a,P}$ and $\phi^{z,P}$ are calibrated at 0.99. Calibration is identical across all model specifications.

1.4.3. Parameter Estimates

Exploiting the RW-MH sampling algorithm, we perform Bayesian estimation of the deep parameters for all model specifications. In the baseline model, the set of the estimated

Table 1.2. Calibration

Parameter	Value	Description
β	0.99	Subjective discount factor;
α	0.3	Steady state share of capital;
δ_0	0.025	Steady state depreciation rate;
u_{ss}	1	Steady state value of capital utilization rate;
μ^x	1.00265	Steady state growth rate of labor-augmenting technology;
μ^a	0.9957	Steady state growth rate of investment-specific technology;
G_{ss}/Y_{ss}	0.2	Steady state share of government expenditure in output;
$\phi^{z,P}$	0.99	Auto-regression coefficient of the permanent component z_t^P ;
$\phi^{x,P}$	0.99	Auto-regression coefficient of the permanent component $\widehat{\mu_t^{x,P}}$;
$\phi^{a,P}$	0.99	Auto-regression coefficient of the permanent component $\widehat{\mu_t^{a,P}}$.

parameters, Θ , is given by

$$\Theta = \begin{pmatrix} \gamma, \kappa, b_c, b_l, \chi, \delta_2 \rho^{xg} \\ \phi_1^{x,T}, \phi_2^{x,T}, \phi_1^{a,T}, \phi_2^{a,T}, \phi_1^{z,T}, \phi_2^{z,T}, \phi_1^g, \phi_2^g, \phi_1^{z^I}, \phi_2^{z^I}, \phi_1^\xi, \phi_2^\xi \\ \sigma_{x,P}, \sigma_{x,T}, \sigma_{a,P}, \sigma_{a,T}, \sigma_{z,P}, \sigma_{z,T}, \sigma_g, \sigma_{z^I}, \sigma_\xi \end{pmatrix}.$$

The first row of Θ consists of the non-calibrated economic parameters; the second row includes the auto-regression coefficients governing the transitional dynamics of the exogenous variables; and the third row incorporates the standard deviation of the corresponding economic shocks.

Table 1.3 displays the prior distribution of the estimated parameters and reports the statistics characterizing their posterior distribution under the baseline model.⁴ In general, we employ flat priors so that the posterior is primarily determined by the likelihood of the data. In the baseline model, the mean estimate of the IES parameter γ is 1.7483, which is consistent with the findings of a large body of the real business cycle literature. Second, the posterior mean of b_c and b_l are 0.7377 and 0.8931, respectively. These estimates imply relatively high degree of habit formation in consumption and leisure. And the posterior mean of δ_2 , the parameter that governs the convexity of the depreciation rate function, is 0.1113, which further implies that the elasticity of capital utilization to the rental rate of capital is approximately 0.6.

Overall, our estimates of the deep economic parameters under the baseline model are consistent with findings reported in the real business cycle literature. Compared with Schmitt-Grohé and Uribe (2012) and Avdjiev (2009) whose empirical practice is based on similar theoretical framework, our estimation achieves non-trivial improvements primarily along

⁴Parameter estimates for alternative model specifications are reported in Table A.4 - A.6 in Appendix A.4. Across all model specifications, the prior distribution of the structural parameters is kept almost identical. The prior distribution in specification 4 is specified with minor difference to avoid the non-positivity of the Hessian matrix arising from estimating the posterior mode.

Table 1.3. Bayesian Estimation of Structural Parameters : Baseline Model

Parameter	Prior			Posterior				
	Distribution	Mean	Std.	Mean	Median	Std.	Percentile 10%	Percentile 90%
γ	Gamma	2	1	1.7483	1.7556	0.0375	1.6952	1.8000
κ	Gamma	4	2	3.7927	3.7974	0.0465	3.7496	3.8325
b_c	Beta	0.5	0.2	0.5679	0.5676	0.0021	0.5633	0.5732
b_l	Beta	0.5	0.2	0.7211	0.7210	0.0036	0.7201	0.7224
χ	Gamma	4	2	3.2202	3.2163	0.0280	3.1612	3.2887
δ_2	Gamma	0.1	0.05	0.1113	0.1112	0.0005	0.1110	0.1117
$\phi_1^{x,T}$	Beta	0.6	0.3	0.4047	0.4050	0.0036	0.4014	0.4074
$\phi_2^{x,T}$	Beta	0.2	0.1	0.2085	0.2085	0.0010	0.2080	0.2089
$\phi_1^{a,T}$	Beta	0.6	0.3	0.5439	0.5438	0.0019	0.5407	0.5470
$\phi_2^{a,T}$	Beta	0.2	0.1	0.1417	0.1415	0.0011	0.1409	0.1426
$\phi_1^{z,T}$	Beta	0.6	0.3	0.7731	0.7728	0.0134	0.7716	0.7748
$\phi_2^{z,T}$	Beta	0.2	0.1	0.2268	0.2270	0.0026	0.2251	0.2283
ϕ_1^ξ	Beta	0.6	0.3	0.8420	0.8420	0.0060	0.8406	0.8435
ϕ_2^ξ	Beta	0.2	0.1	0.1562	0.1563	0.0033	0.1551	0.1572
$\phi_1^{z^I}$	Beta	0.6	0.3	0.5767	0.5763	0.0058	0.5736	0.5807
$\phi_2^{z^I}$	Beta	0.2	0.1	0.3031	0.3031	0.0033	0.3023	0.3037
ϕ_1^g	Beta	0.6	0.3	0.7025	0.7029	0.0047	0.6958	0.7099
ϕ_2^g	Beta	0.2	0.1	0.1948	0.1948	0.0009	0.1933	0.1963
ρ^{xg}	Beta	0.7	0.3	0.3575	0.3575	0.0141	0.3556	0.3592
$\sigma_{x,P}$	I-G	0.02	Inf.	0.0033	0.0033	0.0004	0.0028	0.0037
$\sigma_{x,T}$	I-G	0.1	Inf.	0.0253	0.0253	0.0033	0.0242	0.0264
$\sigma_{a,P}$	I-G	0.02	Inf.	0.0079	0.0079	0.0006	0.0074	0.0085
$\sigma_{a,T}$	I-G	0.1	Inf.	0.0704	0.0713	0.0024	0.0645	0.0738
$\sigma_{z,P}$	I-G	0.02	Inf.	0.0168	0.0169	0.0028	0.0155	0.0181
$\sigma_{z,T}$	I-G	0.1	Inf.	0.0119	0.0119	0.0009	0.0118	0.0122
σ_ξ	I-G	0.1	Inf.	0.0641	0.0640	0.0054	0.0620	0.0665
σ_{z^I}	I-G	0.1	Inf.	0.1394	0.1396	0.0074	0.1316	0.1468
σ_g	I-G	0.1	Inf.	0.0294	0.0294	0.1058	0.0277	0.0312

Note: "I-G" denotes Inverse-Gamma distribution.

two dimensions.⁵ First, Avdjiev (2009) finds substantially lower degree of habit formation in consumption and leisure, which is inconsistent with previous findings. Second, the posterior mean estimates of the Frisch elasticity parameter (χ) and the parameter governing the investment adjustment cost (κ) are 3.2202 and 3.7927, respectively. These values are closer to the generally agreed values in the real business cycle literature than those reported in Schmitt-Grohé and Uribe (2012).

As displayed in Table A.4 - A.6 in Appendix A.4, estimates of the economic parameters are robust to alternative specifications on the exogenous processes. Noticeable differences are discussed as follows. First, the posterior mean of γ under specifications 2 and 4 is around 1.35, which is slightly lower than its baseline counterpart, and the mean estimate under specification 3 is 2.2855, which is marginally higher than 2, the widely accepted upper bound of the IES parameter. Across all model specifications, however, the IES estimates are greater than 1, indicating that, upon the arrival of macroeconomic shocks, the income effect dominates the substitution effect. Second, eliminating the persistent long-run components in labor-augmenting technology and investment-specific technology, specification 4 yields lower estimates of the habit formation parameters than the alternatives. And the posterior mean of δ_2 is only one-quarter as large as those under the alternative models. Finally, while not contradicting the empirical evidence borrowed from the real business cycle literature, the posterior estimates of χ and κ across four model specifications do not exhibit any consistent pattern.

For the parameters regulating the stochastic processes of the exogenous variables, first, the posterior mean estimates of $\sigma_{x,T}$, $\sigma_{z,P}$, $\sigma_{z,T}$, $\sigma_{z,I}$ and σ_g are almost identical across all model specifications. Second, specification 3 yields the mean estimate of σ_ξ twice as large as those under the alternatives. In addition, under specification 2 and 3, the posterior mean of $\sigma_{a,T}$ is around 0.03, which is twice as large as that under specification 4, and in the meantime even less than one half of its baseline counterpart. For other volatility

⁵A major difference between our model and those of Schmitt-Grohé and Uribe (2012) and Avdjiev (2009) is that we do not consider anticipated shocks.

parameters, namely $\sigma_{x,P}$ and $\sigma_{a,P}$, it is found that: (1) under specification 2, where the permanent component of labor-augmenting technology is eliminated, the estimated volatility of shocks to long-run investment-specific technology growth ($\sigma_{a,P}$) is twice as much as the one under the baseline model; and (2) under specification 3, where the long-run component of investment-specific technology is excluded, the estimated volatility of shocks to long-run labor-augmenting technology growth ($\sigma_{x,P}$) is also twice as large as its baseline counterpart.

1.5. Variance Decomposition and Model Comparison

1.5.1. Unconditional Variance Decomposition

So as to identify the major sources of fluctuations in aggregate variables and asset prices, in this subsection, we perform unconditional variance decomposition and attempt to quantify the share of the predicted error variance of the seven observables traceable to each of the macroeconomic shocks. Decomposition statistics under the baseline model are reported in Table 1.4, and findings under the alternative models are presented in Appendix A.4.

First, unconditional variance decomposition highlights the importance of economic shocks to the permanent components of technology growth to accounting for the business cycles. The innovations to the permanent components of total factor productivity growth, labor-augmenting technology growth, and investment-specific technology growth jointly explain 35% - 40% of the predicted error variance of output growth, investment growth, the share of consumption in total output, and hours. While merely 16% of the variation in government spending to output ratio is due to $\epsilon_t^{x,P}$, $\epsilon_t^{a,P}$ and $\epsilon_t^{z,P}$, these shocks explain 49% of the predicted error variance of the risk-free rate. In addition, it is more or less surprising to see that the shocks to persistent long-run technology growth jointly account for almost 86% of the predicted error variance of the share of total market values in output, whereas the contribution of innovations to the transitory components of technology growth is only 14%.

Second, the decomposition statistics shed light on the identification of the key driving forces of the business cycles, and indicates that previous studies might have underestimated

the importance of investment-related technology shocks. As shown in Table 1.4, shocks to the permanent and the transitory components of the investment-specific technology growth explain the largest fraction of fluctuations in output growth, investment growth and the risk-free rate. Shocks to the permanent component of the investment-specific technology growth, $\epsilon_t^{a,P}$, are also the second important predictor of variation in consumption-output ratio. While the most important contributor to variation in $\log(C_t/Y_t)$ is the shocks to the transitory component of total factor productivity growth $\epsilon_t^{z,T}$, its quantified contribution is merely marginally higher than that of $\epsilon_t^{a,P}$. However, $\epsilon_t^{z,T}$ is indeed one of the most important factor explaining the predicted error variance of consumption to output ratio and government spending to output ratio. For the asset market variable, it is once again surprising to find that nearly 88% of the variation in the ratio of total market values to output is accounted for by innovations to the permanent and the transitory components of investment-specific technology growth. Except for shocks to the transitory component of labor-augmenting technology growth, other macroeconomic innovations play literally no role in explaining asset price movements.

Compared with the evidence from the existing literature, our unconditional variance decomposition is consistent with the findings of Justiniano, Primiceri and Tambalotti (2008), which argues that shocks to investment-specific technology play a central role in driving aggregate fluctuations. However, these results are in sharp contrast to those of Avdjiev (2009) and Schmitt-Grohé and Uribe (2012). The apparent discrepancy might arise from different assumptions made on the underlying modeled economy (such as preference specifications, incorporation of certain variables and sectors, and so forth), but usually seems not quantitatively traceable.⁶ As shown in the rest of this subsection, even small changes in the specification on the stochastic processes of certain exogenous variables, while keeping everything else constant, are likely to lead to remarkably distinct model implications.

⁶For instance, while Avdjiev (2009) employs similar theoretical framework to ours, several important variables, such as government spending and preference shocks, are missing. Schmitt-Grohé and Uribe (2012) employ the Jamovich-Rebelo utility function and take into account both anticipated and unanticipated shocks.

Table 1.4. Unconditional Variance Decomposition : Baseline

	Shock	g^Y	g^{AI}	$\log\left(\frac{C}{Y}\right)$	$\log\left(\frac{G}{Y}\right)$	$\log\left(\frac{V}{Y}\right)$	H	R^{rf}
Persistent	$\epsilon^{x,P}$	0.1533	0.1046	0.0292	0.0339	0.1102	0.0295	0.1442
	$\epsilon^{a,P}$	0.2441	0.2869	0.3327	0.1123	0.7501	0.3176	0.2128
	$\epsilon^{z,P}$	0.0070	0.0093	0.0040	0.0150	0.0007	0.0073	0.1326
Sum		0.4044	0.4008	0.3659	0.1612	0.8610	0.3544	0.4896
Transitory	$\epsilon^{x,T}$	0.1450	0.0371	0.0049	0.0050	0.0048	0.0036	0.0350
	$\epsilon^{a,T}$	0.3797	0.3850	0.1294	0.0925	0.1270	0.1021	0.2746
	$\epsilon^{z,T}$	0.0020	0.0028	0.3685	0.5037	0.0020	0.5118	0.1534
	ϵ^ξ	0.0054	0.0010	0.1240	0.1940	0.0004	0.0241	0.0050
	ϵ^{z^I}	0.0565	0.1727	0.0072	0.0322	0.0048	0.0039	0.0125
	ϵ^g	0.0071	0.0006	0.0002	0.0113	0.0001	0.0001	0.0300
Sum		0.5957	0.5992	0.6342	0.8387	0.1391	0.6456	0.5105

Under specification 2, where the persistent component of labor-augmenting technology is omitted, innovations to the persistent long-run growth of total factor productivity and investment-specific technology remain to be the major drivers of aggregate fluctuations. And shocks to the permanent and the transitory components of investment-specific technology growth once again contribute the largest fraction to accounting for the predicted error variance of key aggregate variables. However, the importance of permanent and transitory total factor productivity shocks diminishes in a relatively significant manner. While the contribution of total factor productivity shocks to risk-free rate increases by more than 30%, the share of variation in consumption and in government expenditures explained by $\epsilon_t^{z,P}$ and $\epsilon_t^{z,T}$ declines sharply to nearly 0.5%.

For specification 3, where persistent long-run investment-specific technology growth is excluded, it is found that variation in output growth, investment growth, consumption and total market values are primarily driven by shocks to the permanent and the transitory components of labor-augmenting technology growth. Shocks to government expenditures explain the largest share of the government expenditures to output ratio, and the second largest share

of variation in consumption-output ratio. The effect of shocks to other exogenous variables is quantitatively close to zero. Under specification 4, where total factor productivity growth is the only exogenous variable consisting of a permanent component, shocks to total factor productivity growth and to labor-augmenting technology growth explain 48% and 28% of the variation in output growth, respectively; innovations to transitory investment-specific technology growth are the major contributors to fluctuations in investment, consumption and total market values; shocks to z_t^I explain the second largest share of the predicted error variance of investment growth; preference shocks explain 78% of the variation in hours; and shocks to government expenditures literally plays no role.

Comparing the baseline variance decomposition with those under the three alternatives unambiguously demonstrates that the identification of the key drivers of aggregate fluctuations hinges critically upon researchers' specification on the structure of the exogenous processes. The relatively high sensitivity of decomposition statistics to small changes in the assumption on the exogenous processes is not a trivial issue, because the true data generating processes of the exogenous variables are usually unobservable, and making arbitrary assumption on their unobserved structure can potentially lead to biased model implications. To minimize the impact of misspecification on model inference, we exploit Bayesian model comparison technique to identify the model specification that is mostly supported by the data. This issue is discussed in section [1.5.3](#).

1.5.2. Conditional Variance Decomposition

In addition to unconditional variance decomposition, we exploit conditional variance decomposition to investigate the key drivers of aggregate fluctuations from a dynamic perspective. First, as displayed in Appendix [A.5](#), decomposition results under the baseline model indicate that both the long-run and the short-run forecasting error variance of output growth is primarily explained by shocks to the transitory components of labor-augmenting and investment-specific technology growth; the contribution of shocks to the permanent components of these two technology growth rates is quantitatively small at short forecasting

horizons, but increases dramatically in the long-run. In particular, for investment-specific technology growth, the contribution of shocks to its transitory component dominates the contribution of those to its permanent component in the short-run, but becomes dominated at long forecasting horizons. For macroeconomic innovations to the technology rate of transforming investment goods into capital, ϵ^{z^I} , their contribution to the variation in output growth, investment growth, and the shares of consumption and government expenditures in output is non-negligible, but diminishes as the forecasting horizon increases. Similar to ϵ^{z^I} , government expenditure shocks induce more than 50% of the variation in government expenditures to output ratio in the short-run, but the contribution falls sharply at longer forecasting horizons and eventually becomes zero.

When the permanent component of investment-specific technology growth is omitted, model implication under specification 2 is partly consistent with its baseline counterpart. Noticeable distinction, however, lies in the fact that a set of exogenous shocks that play trivial roles in the baseline model become significant contributors to inducing short-run aggregate fluctuations. For instance, at short forecasting horizons, shocks to the permanent component of total factor productivity explain a large fraction of the forecasting error variance of output; the contribution of preference shocks to inducing short-run fluctuations in consumption-output ratio is almost identical to that of shocks to the permanent component of investment-specific technology growth; and over 70% of the short-run variation in government expenditures to output ratio can be attributed to government expenditure shocks.

It is worth mentioning that the findings of the conditional variance decomposition under each model specification further confirms that inference about the major driving forces of the business cycles, regardless of the forecasting horizons, hinges critically upon the specification on the structure of the exogenous processes. Presumption on the unobserved stochastic processes can potentially lead to biased model implications. Therefore, evaluating a set of competing alternatives based on the likelihood of the data seems necessary to mitigate the impact of misspecification.

1.5.3. Bayesian Model Comparison

In Appendix A.6, we plot the series of data on the seven observed variables along with their smoothed counterparts. In general, all model specifications fit the observables reasonably well. Therefore, the statistics of the goodness-of-fit can hardly help to assess the relative plausibility of these model specifications. Hence, we compute the Bayesian odd ratio of each model to figure out which specification is mostly favored by the likelihood of the data. Since it is difficult to verify the existence of persistent long-run growth in labor-augmenting technology and investment-specific technology *a priori*, equal priors are assigned to the baseline model and the three alternatives. We find that specification 2 and the baseline model yield the highest and the 2nd highest marginal density, respectively; and under specification 3, where the variance decomposition implications are dramatically distinct from its competing alternatives, has the lowest marginal density. In addition, even though the marginal density of the baseline model is slightly lower than that of specification 2, the odd ratio test overwhelmingly supports the specification that labor-augmenting technology growth does not consist of a permanent component. Given that the odd ratio under specification 2 is surprisingly large, the estimated posterior probability is 1. Therefore, these findings indicate that previous studies might have overlooked persistent long-run investment-specific technology growth, and thus, fail to sufficiently capture its effect on driving the business cycles.

1.6. Aggregate Dynamics and Asset Price Movements

The remaining task of this study is to investigate how aggregate variables make dynamic adjustments to the macroeconomic innovations. In Appendix A.7, Figure A.13 displays the impulse response functions of the investigated variables to a one standard deviation (S.D. hereafter) negative shock to the permanent component of investment-specific technology growth under specification 2. It is worth mentioning that a shock of positive value to investment-specific technology growth is interpreted as a negative investment-specific technology shock. This is because, in a decentralized market, the variable A_t measures the relative price of investment goods, and hence, an increase in A_t implies lower efficiency of

Table 1.5. Bayesian Model Comparison

	Baseline	Specification 2	Specification 3	Specification 4
Prior Probability	0.25	0.25	0.25	0.25
Log Marginal Density	3512.13	3575.87	3355.44	3396.71
Odd Ratio	1.00	4.84×10^{27}	0.00	0.00
Posterior Probability	0.00	1.00	0.00	0.00

investment-specific technology. When a one S.D. negative shock of $\epsilon_t^{a,P}$ is imposed, investment initially increases, but starts to decrease after 10 quarters, and eventually becomes negative at long forecasting horizons. While seemingly perplexing, the fact that investment does not fall immediately in response to a negative investment-productivity shock is not difficult to understand. It is because, when the negative shock of $\epsilon_t^{a,P}$ becomes realized, forward-looking economic agent is aware that the shock would induce permanent changes in investment-specific technology growth. Given its persistent nature, the representative household expects that the relative price of investment would continue to rise in the future. As a consequence, the agent would tend to revise her investment plan by investing more immediately (when the price of investment is relatively cheap) and investing less in the future (when the price of investment is relatively expensive).

Second, Figure A.13 suggests that consumption and leisure increase in response to $\epsilon_t^{a,P}$. Given that negative investment-specific technology shocks induce higher relative price of investment goods with respect to consumption goods, the representative household would tend to substitute away from investment and simply choose to consume more. In addition, once exposed to a negative investment-specific technology shock, the representative firm also has incentives to re-optimize its production plan. Figure A.13 shows that the utilization rate of capital falls as the relative price of investment increases. The magnitude of such

adjustment in capital utilization is large enough to offset the effect of reduction in investment, and consequently leads to an increase in effective capital stock. Since higher level of effective capital stock reduces the marginal productivity of capital, the relative price of capital to labor also decreases, resulting in less demand for labor. Therefore, leisure declines in response to a negative investment-specific technology shock, the impact of which can be potentially large enough to induce decreases in output at business cycle frequencies. For asset price movements, it is somehow surprising to find that a negative investment-specific technology shock increases total market values.⁷ A potential explanation, however, could be that, even though output falls and risk free rate rises in response to a negative investment-productivity shock, investment decreases by more, which generates a sufficiently large dividend effect such that the expected sum of future dividend payment increases enough to offset the risk-free rate effect and eventually leads to an increase in total market values.

For a one S.D. $\epsilon_t^{a,T}$ shock, Figure A.14 indicates that the pattern of the impulse responses of the investigated aggregate variables is largely similar to that in the case where $\epsilon_t^{a,P}$ is imposed. The similarity is unambiguously due to the fact both $\epsilon_t^{a,P}$ and $\epsilon_t^{a,T}$ affect the economy through the same channel (decreasing the growth rate of investment-specific technology). Nevertheless, we observe that the dynamic responses of all macroeconomic variables induced by $\epsilon_t^{a,T}$ are not only less persistent but also in much smaller magnitude. These findings are consistent with the definition of these two types of shocks.

Under specification 2, the other exogenous variable assumed to consist of a permanent component is the total factor productivity growth. Figure A.15 shows that, the arrival of a one S.D. $\epsilon_t^{z,P}$ shock has an immediate positive effect on investment, but such impact diminishes gradually at longer forecasting horizons. Capital utilization rate falls by 3% instantaneously, and returns to its original level in a few quarters afterward. And a higher level of investment, combined with declined utilization rate, increases capital stock. In addition,

⁷Admittedly, our theoretical framework, as well as other standard business cycle models, might fail to provide fully adequate explanations to fluctuations in asset prices. Because, instead of explicitly modeling the stochastic discount factor, these types of analytical framework usually assume that asset price movements are purely driven by the risk-free rate and future cash-flows.

Figure A.15 suggests that a positive total factor productivity shock tends to induce persistent increases in consumption and leisure, whereas its effect on the risk-free rate is not only negative but also short-lived. In terms of production, a positive $\epsilon_t^{z,P}$ shock leads to persistent increases in total output over time. Conditional on higher levels of output and investment, our model predicts that their net effect on future dividend payment is negative. As a consequence, the interest rate effect slightly dominates the dividend effect, and thus, induces extremely short-lived increases in total market values. Compared with $\epsilon_t^{z,P}$, a one S.D. $\epsilon_t^{z,T}$ shock induces similar pattern of dynamic adjustments of the macroeconomic variables, but its effect is not only short-lived but also smaller in magnitude. In addition, it is worth pointing out that the magnitude of changes in macroeconomic variables induced by total factor productivity shocks is merely one-tenth as large as those generated by the investment-specific technology shocks, which further confirms that investment-specific technology shock is very likely to be the major driving force of the business cycles.

For a one S.D. shock to the transitory component of labor-augmenting technology growth, it induces instantaneous decreases in investment and the capital utilization rate. But its negative impact on these two variables becomes positive after 5 quarters, and eventually dies out at longer forecasting horizons. The net effect of investment and capital utilization rate responses on capital stock is initially negative and diminishes as the forecasting horizon increases. Upon the arrival of $\epsilon_t^{x,T}$, our model predicts that the representative household reduces her consumption by fairly small amount. While hours increase in response to a positive labor-augmenting technology shock, short-run effective capital stock does decrease by a sufficiently large amount so that total output falls in the first 10 quarters. When effective capital stock resumes to its original level and hours reach its peak in the 10th quarter, changes in output become positive. However, the positive effect of labor-augmenting technology shock on output is not long-lasting and eventually dies out at longer forecasting horizons. In addition, it is found that a positive labor-augmenting technology shock reduces dividends and results in a hump-shaped dynamic path. In the meanwhile, interest rate falls by 1.5% immediately after the shock is imposed, and goes back to its original level in 10 periods.

Combined with the dynamic response of total market values, these pieces of evidence indicate that positive labor-augmenting technology shocks tend to increase total market values in the short-run through the interest rate channel, and reduce total market values in the long-run via the dividend channel.

As suggested in the literature, another potentially important source of aggregate fluctuations is the shock to the marginal efficiency of investment. Under our framework, investment, capital utilization and hours all respond positively upon the arrival of a positive one S.D. ϵ^{z^I} shock, and gradually fall below their original levels after 15 forecasting periods. In the long-run, ϵ^{z^I} produces negative impact on investment, capital utilization rate and hours. However, the magnitude of its long-run negative effect is smaller than that of its initial positive effect. In addition, capital stock responds positively to a positive ϵ^{z^I} shock throughout the forecasting horizons, and the dynamics of total output is similar to those of investment and hours. It is also found that the dynamic path of firm's profit is close to the mirror image of that of investment (and output). As risk-free rate only responds positively to the shock to z^I during the initial periods, impulse responses of total market values almost replicate the dynamic path of dividend payment.

For other macroeconomic innovations, first, we find that, while relatively small in magnitude, a one S.D. preference shock is able to induce permanent changes in consumption, labor supply, investment, output and total market values. Second, the effect of government spending shocks on the aggregate variables is instantaneous, but not long-lasting. Finally, it is worth pointing out that several perplexing phenomena are identified in the impulse responses analysis using other model specifications. For instance, under the baseline model, shocks to the transitory component of total factor productivity seem to induce permanent changes in a subset of the aggregate variables; and total market values fall throughout the entire forecasting periods responding to positive shocks to the permanent and the transitory components of labor-augmenting technology.⁸

⁸These anomalous findings are not reported in the paper. However, they are available upon request.

1.7. Conclusion

In this study, we develop a neoclassical growth model and seek to investigate the major driving forces of the business cycles. So as not to rule out the possibility that the growth rates of total factor productivity, labor-augmenting technology and investment-specific technology consist of persistent long-run components, we specify a baseline model along with three competing alternatives through varying the assumption on the structures of the stochastic processes of these exogenous variables, and exploit Bayesian inference methods to assess their relative plausibility.

Quantitative analysis based on variance decomposition, Bayesian odd ratio test and impulse response functions indicates that the identification of the key drivers of the aggregate fluctuations is heavily contingent upon researchers' assumption on the structure of the exogenous processes. In particular, empirical evidence suggests that previous studies might have overlooked the persistence embedded in the process of investment-specific technology growth, and underestimate its importance to driving the business cycles. According to our findings, shocks to investment-specific technology account for a vast majority of the short-run and the long-run predicted error variance of output growth, investment growth, the share of government expenditures in output, hours, and total market values to output ratio; preference shocks explain over 50% of the short-run fluctuations in the consumption-output ratio; a large fraction of the short-run fluctuations in hours is attributable to shocks to the permanent component of labor-augmenting technology; and government spending shocks are only important to explaining the movements in the government expenditures to output ratio at short forecasting horizons.

Compared with previous empirical studies which employ similar theoretical framework, parameter estimation in this paper achieves noticeable improvements along a few dimensions. However, this study has several limitations needed to be overcome in future work. First, considering that it would be difficult to infer a large variance-covariance matrix of the exogenous shocks from a relatively small sample with merely seven observables, we simply assume that the macroeconomic innovations are orthogonal to each other. Admittedly, this assumption

seem not quite plausible. Relaxing the orthogonality assumption might help to resolve several perplexing phenomena reported in the impulse responses analysis, but needs to be based on more reasonable identification (or estimation) schemes capable of accurately extracting information from the medium frequency data. In addition, similar to other neoclassical models using standard non-recursive preference specifications, the theoretical framework adopted in this paper seems not able to adequately capture the movements in asset prices. Modern asset pricing literature argues that asset price movements are primarily driven by variation in the stochastic discount factor. The long-run risks literature, which achieves remarkable improvements on replicating the asset pricing facts in partial equilibrium framework, highlight the importance of long-run consumption growth and its conditional volatility to resolving several asset pricing puzzles. In contrast, macroeconomic innovations in our modeled economy can only affect asset prices through the interest rate channel and the dividend channel. To adequately capture the interactions between asset prices and the market fundamentals, it seems necessary to incorporate recursive (e.g. Epstein-Zin) preference specifications and properly model persistent long-run consumption growth under the general equilibrium framework.

CHAPTER 2

WHAT'S DRIVING THE LOW-FREQUENCY MOVEMENTS IN STOCK PRICES? A CONSUMPTION-BASED ASSET PRICING MODEL WITH RECURSIVE PREFERENCES

2.1. Introduction

The macro-finance literature seeks to establish the linkages between macroeconomic activities and asset price movements, and further identify the macroeconomic innovations that are important to accounting for the asset pricing facts. Exploiting the standard non-recursive and homogeneous preference specifications, earlier consumption-based asset pricing studies propose a few channels through which macroeconomic shocks to aggregate consumption are translated into the primary sources of asset market fluctuations, and have been shown successful in replicating a few key features of the U.S. data in the simulated environments. Prominent examples of these streams of work include the habit-formation model (Campbell and Cochrane (1999)), the rare disaster model (Rietz (1988), Barro (2006), Gabaix (2010), and Wachter (2011)), and so forth.

Inspired by Epstein and Zin (1991) and Weil (1989), recent macro-finance literature explores the asset pricing implications under models with recursive preferences, a type of preference specifications that allows for a separation between the Intertemporal Elasticity of Substitution (hereafter, IES) and the risk aversion coefficient. The long-run risks literature, motivated by Bansal and Yaron (2004), represents a notable stream of studies along this line of effort. Under the standard long-run risks framework, consumption growth not only is subject to macroeconomic shocks at the business cycle frequencies, but also consists of a predictable long-run component; and asset market variables (such as the price-dividend ratio and the market returns) fluctuate in response to changes in long-run consumption growth

and its conditional volatility. Bansal and Yaron (2004), as well as a series of subsequent studies (such as Bansal, Khatchatrian and Yaron (2005), Bansal, Dittmar and Kiku (2009), Bansal, Kiku and Yaron (2012)), provide strong empirical evidence indicating that the long-run risks model is capable of matching a variety of sample moments associated with the price-dividend ratio, the dividend growth, and the market returns. These findings highlight the importance of the long-run consumption growth channel to resolving the asset pricing puzzles.

While shedding light on our understanding of the macroeconomic sources of asset market fluctuations from a new perspective, the long-run risks literature poses a few questions that remain to be thoroughly investigated. First, it is widely known that the long-run risks model relies heavily on the assumption that the IES parameter is greater than 1. This assumption, however, is strongly against the empirical evidence found using macro- and micro-level data. Bansal, Kiku and Yaron (2012) argue that using an IES parameter that is greater than unity is not necessarily implausible, because the IES estimates reported in studies that impose homoscedastic variance assumptions are likely to be downward-biased. As pointed out by Beeler and Campbell (2012), however, with an IES parameter greater than 1, the long-run risks model tends to predict strong co-movements between aggregate consumption growth and the short-term interest rate, a pattern that is not observed in the U.S. data.

Second, previous long-run risks studies usually employ moment-matching methods to demonstrate the model's ability to rationalize the data. Due to the lack of studies that attempt to replicate the dynamic paths of the cash flow and the return variables, it is still unclear whether the long-run risks model can adequately account for the asset market phenomena. A major difficulty that complicates this fitting task lies in extracting the latent state variables from the low-frequency data. Earlier work along this dimension (such as Bansal, Kiku and Yaron (2007, 2012)) seeks to recover the latent states using either constrained linear regressions or grid-searches. Nevertheless, the aforementioned approaches in general overlook the non-linear structure of the exogenous processes, and thus, seem insufficient to accurately handle the non-linear tracking problems.

In addition, standard long-run risks framework attributes asset market fluctuations solely to variation in long-run consumption growth. In contrast, recent asset pricing literature emphasizes the significance of potential driving forces that are not directly related to aggregate consumption. Among others, studies that investigate the relationship between time-preferences and asset prices have drawn increasing attention.¹ Intuitively, shocks to time-preferences (or tastes) unavoidably affect agents' willingness to substitute between consumption and investment. As a consequence, not only the asset prices would need to respond to changes in demand for equity assets, but also the required equity returns should compensate the asset holders for bearing the valuation risks.² Since little research effort has been made to augment the long-run risks framework with additional channels, it remains unsettled what asset pricing implications the long-run risks model would deliver once non-consumption-related macroeconomic innovations are present.

Attempting to explicitly address the aforementioned questions, this study follows Bansal and Yaron (2004), and Albuquerque, Eichenbaum and Rebelo (2013), and proposes a consumption-based asset pricing model with recursive preferences. Extending the standard long-run risks framework, our theoretical model not only retains the long-run consumption growth channel in the Bansal and Yaron (2004) fashion, but also features a time-preference shocks channel through which innovations to agents' time-preferences are translated into the determinants of asset prices. So as to capture the long-run and the valuation risks embedded in the low-frequency movements in consumption growth, dividend growth, and time-preference shocks, we generalize the specification of the exogenous processes to allow for time-varying volatilities. In our modeled economy, asset market variables fluctuate in response to long-run consumption growth, time-preference shocks and their respective volatilities; and expected equity premium reflects the market compensation for households' exposure to consumption growth uncertainty and valuation risks. In particular, the model predicts that positive time-

¹Previous asset pricing studies that evaluate the effect of time-preference shocks or taste shocks on asset market variables include Garber and King (1983), Campbell (1988), and Albuquerque, Eichenbaum and Rebelo (2013).

²The concept of valuation risks was introduced in Albuquerque, Eichenbaum and Rebelo (2013). Valuation risks measure to what extent the agent is uncertain about how to discount current versus future cash flows.

preference shocks increase the price-dividend ratio, and in the meantime reduce the risk-free rate. Moreover, while not directly implied by the theoretical model, we provide evidence that the responses of risk-free rate and price-dividend ratio to fluctuating economic risks would depend on the nature of the risks. According to our findings, risk-free rate and price-dividend ratio rise in response to increased long-run consumption growth uncertainty, and fall as a consequence of higher valuation risks.

To assess the model’s ability to account for the data, this study develops a two-step empirical approach to estimate the economic parameters and the latent states. To be specific, the first step of our empirical practice involves estimation of the deep parameters using the Generalized Method of Moments (hereafter, GMM), where time-aggregation is taken into account to correct the information loss resulting from the potential mismatch between agents’ decision making interval and our sampling frequency.³ In the second step, we derive a state-space representation of the equilibrium conditions, and then infer the latent state variables by approximating their probability densities using the particle filter and the particle smoother.⁴ We find that, in general, the proposed model yields quite moderate estimates of the IES parameter and the risk aversion coefficient, and in the meantime, is able to explain the joint dynamics of the price-dividend ratio, the risk-free rate, the market returns and the realized equity premium.

Based on various experiments on model specifications and tests, this study provides strong empirical evidence demonstrating the importance of long-run consumption growth and time-preference shocks to understanding the asset price movements over the business cycles. Historical decomposition suggests that the fluctuations in price-dividend ratio and market returns are overwhelmingly due to movements in long-run consumption growth, whereas time-preference shocks and valuation risks are crucial determinants of the risk-free rate. In

³Earlier studies that consider time-aggregation include Hansen and Sargent (1983), Heaton (1995), and Bansal, Kiku and Yaron (2012).

⁴In a recent study, Schorfheide, Song and Yaron (2014) propose a mixed-frequency Bayesian approach to estimate the long-run risks model, in which the likelihood function is also approximated using the particle filter. While their study and ours share the same spirit, there are a few key differences. Further discussion is provided in Section 2.4.

addition, we find that excluding any of these two channels substantially weakens the model’s ability to jointly fit the data. In particular, we show that the standard long-run risks model by itself cannot adequately track the risk-free rate; and absent the long-run consumption growth channel, the “time-preference shocks only” model fails to capture the persistence of the price-dividend ratio dynamics. In response to the question raised in Beeler and Campbell (2012), we find that, in the presence of the time-preference shocks channel, our model does not imply strong co-movements between consumption growth and the risk-free rate when IES is greater than 1. Moreover, according to our experiments, the contribution of time-preference shocks to asset market fluctuations increases as the time-preference shocks process becomes more persistent. However, we show that persistent time-preference shocks, such as those reported in Albuquerque, Eichenbaum and Rebelo (2013), is not supported by the U.S. data.

The rest of this study is organized as follows. In section 2.2 we introduce the theoretical model and present the solution to the equilibrium conditions. Section 2.3 describes the GMM estimation procedure and reports the parameter estimates. Section 2.4 presents our particle filtering and particle smoothing algorithms, and discusses the empirical findings. And section 2.5 concludes.

2.2. The Model

Consider a simple representative-agent endowment economy proposed in Albuquerque, Eichenbaum and Rebelo (2013). We assume that the representative agent has the Epstein and Zin (1991) and Weil (1989) type recursive preferences. Specifically, the representative agent’s life-time utility V_t is defined as a function of current consumption and expected future utility:

$$V_t = \left[\lambda_t (C_t)^{1-1/\psi} + \delta (V_{t+1}^*)^{1-1/\psi} \right]^{1/(1-1/\psi)}, \quad (2.1)$$

where C_t denotes consumption at time t ; δ is a constant subjective discount factor; and ψ denotes the parameter that governs the intertemporal elasticity of substitution. V_{t+1}^* represents the certainty equivalent of the agent’s life time utility at time $t + 1$, which is

defined as

$$V_{t+1}^* = [E_t(V_{t+1}^{1-\gamma})]^{1/(1-\gamma)}, \quad (2.2)$$

where γ measures the degree of relative risk aversion. In addition, the sequential budget constraint of the representative agent is given by

$$W_{t+1} = (W_t - C_t) R_{c,t+1}. \quad (2.3)$$

In equation (2.3), W_t denotes total wealth, and $R_{c,t+1}$ is the gross return on invested net wealth in period $t + 1$. Compared with the conventional asset pricing models with recursive preferences, the proposed model introduces a time-preference factor λ_t , which captures the weight attached to consumption in each period. We allow λ_t to vary over time. As shown later, the stochastic discount factor (or the pricing kernel) will respond to changes in agent's time-preferences.

Albuquerque, Eichenbaum and Rebelo (2013) show that, for any dividend-paying asset i , its gross return $R_{i,t+1}$ satisfies the following Euler equation:

$$E_t (M_{t+1} R_{i,t+1}) = 1, \quad (2.4)$$

where M_{t+1} denotes the stochastic discount factor, which takes the following form:

$$M_{t+1} = \left(\frac{\lambda_{t+1}}{\lambda_t} \delta \right)^\theta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} (R_{c,t+1})^{\theta-1}. \quad (2.5)$$

In equation (2.5), $\theta \equiv (1 - \gamma)/(1 - 1/\psi)$; and the ratio $(\lambda_{t+1}/\lambda_t)$ interacts with δ in a way such the actual subjective discount factor is affected by the weight attached to consumption in current and future periods. Following Albuquerque, Eichenbaum and Rebelo (2013), we refer to $(\lambda_{t+1}/\lambda_t)$ as time-preference shock.

2.2.1. Exogenous Processes

In this study, we specify the exogenous processes as follows:

$$x_{t+1} = \rho x_t + \varphi_e \sigma_t e_{t+1}, \quad (2.6)$$

$$h_{t+1} = \rho_\lambda h_t + \sigma_{\lambda,t} \varepsilon_{t+1}, \quad (2.7)$$

$$\Delta c_{t+1} = \mu + x_t + \sigma_t \eta_{t+1}, \quad (2.8)$$

$$\Delta d_{t+1} = \mu_d + \phi x_t + \varphi_d \sigma_t u_{t+1}. \quad (2.9)$$

Here $\Delta c_{t+1} \equiv \log(C_{t+1}/C_t)$ denotes consumption growth, with unconditional mean μ ; $\Delta d_{t+1} \equiv \log(D_{t+1}/D_t)$ denotes the growth rate of dividend payment, with unconditional mean μ_d ; $h_{t+1} \equiv \log((\lambda_{t+1}/\lambda_t))$ represents the percentage change in time preferences; and x_t is defined as a small persistent component that governs the long-run behavior of consumption and cash-flow dynamics.⁵ To ensure stationarity, we assume that ρ and ρ_λ , in equations (2.6) and (2.7), lie within the (0, 1) interval.

In order to capture the economic uncertainties embedded in the low-frequency movements in cash flow variables and time-preference shocks, the exogenous processes are generalized to allow for time-varying volatilities. Note that, under constant volatility settings, the model cannot generate a time-varying risk premium.⁶ According to our specification, the processes of x_{t+1} , Δc_{t+1} and Δd_{t+1} share a common volatility factor σ_t , which captures the uncertainty associated with long-run consumption growth; and $\sigma_{\lambda,t}$ in the process of h_t measures the degree that the representative agent is uncertain about how much she would value future cash flows (relative to current cash flows). Following the literature, we refer to σ_t and $\sigma_{\lambda,t}$

⁵In the long-run risks literature, x_t is often referred to as long-run consumption growth.

⁶We solve an alternative version of the model in which the volatilities of the exogenous processes are constant. Even though this alternative model with homoscedastic volatility yields largely similar asset pricing implications, it cannot justify the time-varying feature of risk premium from the theoretical ground. Similar argument can be found in Bansal and Yaron (2004). Our solution to the constant volatility model is not reported in this study, but is available upon request.

as long-run risks and valuation risks, respectively. The law of motion of these volatility processes are specified as

$$\sigma_{t+1}^2 = \sigma_0^2 + v_1(\sigma_t^2 - \sigma_0^2) + \sigma_w w_{t+1}, \quad (2.10)$$

$$\sigma_{\lambda,t+1}^2 = \sigma_\lambda^2 + v_\lambda(\sigma_{\lambda,t}^2 - \sigma_\lambda^2) + \sigma_\pi \pi_{t+1}, \quad (2.11)$$

where σ_0^2 and σ_λ^2 are the unconditional mean of σ_{t+1}^2 and $\sigma_{\lambda,t+1}^2$, respectively; v_1 and v_λ are auto-regression coefficients that regulate the persistence of these two processes; and σ_w and σ_π are constant. Throughout equations (2.6) to (2.11), we assume that the exogenous shocks (namely e_{t+1} , ε_{t+1} , η_{t+1} , u_{t+1} , w_{t+1} and π_{t+1}) are *i.i.d.* standard normal, and are mutually independent.

2.2.2. Model Solution

To derive the solution to the equilibrium conditions, we define the gross return on the consumption asset, $R_{c,t+1}$, and the gross return on the equity asset, $R_{d,t+1}$, as follows,

$$R_{c,t+1} = \frac{P_{c,t+1} + C_{t+1}}{P_{c,t}}, \quad (2.12)$$

$$R_{d,t+1} = \frac{P_{d,t+1} + D_{t+1}}{P_{d,t}}, \quad (2.13)$$

where $P_{c,t+1}$ denotes the price of the consumption asset⁷; $P_{d,t+1}$ is the price of the equity asset; and D_{t+1} denotes the dividend payment. Applying the Campbell and Shiller (1989) approximation yields

$$r_{c,t+1} = k_{0,c} + k_{1,c}z_{c,t+1} - z_{c,t} + \Delta c_{t+1}, \quad (2.14)$$

$$r_{d,t+1} = k_{0,d} + k_{1,d}z_{d,t+1} - z_{d,t} + \Delta d_{t+1}, \quad (2.15)$$

⁷The consumption asset can be interpreted as the representative agent's investment portfolio that delivers consumption good as its dividend payment.

where $z_{c,t} \equiv \log(P_{c,t}/C_t)$ denotes the price-consumption ratio; and $z_{d,t} \equiv \log(P_{d,t}/D_t)$ denotes the price-dividend ratio.⁸ The constant terms $k_{0,j}$ and $k_{1,j}$ ($j = c, d$) are given by

$$k_{0,j} = \log(1 + \exp(\bar{z}_j)) - k_{1,j}\bar{z}_j, \quad (2.16)$$

$$k_{1,j} = \frac{\exp(\bar{z}_j)}{1 + \exp(\bar{z}_j)}, \quad (2.17)$$

where \bar{z}_j is the mean of the sequence $\{z_{j,t}\}_{t=1}^T$.

Given that consumption and dividend growth rates are purely exogenous, to obtain a complete characterization of the equilibrium conditions requires us to pin down the relationship between the state variables and $z_{j,t}$, for $j = c, d$. We conjecture that $z_{c,t}$ and $z_{d,t}$ are linear combinations of x_t , h_t , σ_t^2 , and σ_λ^2 :

$$z_{c,t} = A_0 + A_1x_t + A_2h_t + A_3\sigma_t^2 + A_4\sigma_{\lambda,t}^2, \quad (2.18)$$

$$z_{d,t} = A_{0,d} + A_{1,d}x_t + A_{2,d}h_t + A_{3,d}\sigma_t^2 + A_{4,d}\sigma_{\lambda,t}^2, \quad (2.19)$$

where the A 's are the coefficients to be determined. Rewriting equation (2.5) in logarithm, we have

$$m_{t+1} = \theta \log(\delta) + \theta h_{t+1} - \left(\frac{\theta}{\psi}\right) \Delta c_{t+1} + (\theta - 1) r_{c,t+1}. \quad (2.20)$$

Plugging equations (2.18) and (2.20) into the Euler Equation, we solve for the A 's by matching up the undetermined coefficients. For price-consumption ratio $z_{c,t}$, it can be show that⁹

$$A_1 = \frac{1 - \frac{1}{\psi}}{1 - k_{1,c}\rho},$$

$$A_2 = \frac{\rho_\lambda}{1 - k_{1,c}\rho_\lambda},$$

⁸In the following paragraphs, for any variable S , we use the lower case letter s to denote $\log(S)$.

⁹Detailed derivation of the model solution is provided in Appendix B.1.

$$A_3 = \frac{\theta}{2(1 - k_{1,c}v_1)} \left[\left(1 - \frac{1}{\psi}\right)^2 + (k_{1,c}A_1\varphi_e)^2 \right],$$

$$A_4 = \frac{\theta(1 + k_{1,c}A_2)^2}{2(1 - k_{1,c}v_\lambda)}.$$

According to the solution to the A 's, price-consumption ratio rises in response to higher long-run consumption growth when ψ (IES) is greater than 1; and falls in response to increased long-run risks when ψ and γ are both larger than 1.¹⁰ Here, it is worth noting that the solution to A_1 and A_3 coincides with its counterpart in Bansal and Yaron (2004). This finding indicates that the sensitivity of $z_{c,t}$ to long-run consumption growth and its conditional volatility, under our framework, remains the same as that in the Bansal-Yaron economy, even though now the modeled economy is augmented with time-preference shocks.

Exploring the solution to the coefficients on the components forming the time-preference shocks channel (namely h_t and $\sigma_{\lambda,t}$), first, we find that the price-consumption ratio increases in response to positive time preference shocks, and its sensitivity is increasing in ρ_λ . Recall that h_t measures the percentage change in the weight attached to consumption. Conditional on the persistence of h_t , the representative agent forms the expectation that she would value more of future consumption than current consumption when $h_t > 0$. Being more patient, the agent would tend to reallocate more of the resources to investment assets (by refraining current consumption), and as a consequence, the price-consumption ratio rises. Therefore, as ρ_λ becomes larger, time-preference shocks will induce more sensitive responses of the price-consumption ratio through shifting asset demand. Second, the solution to A_4 indicates that an increase in $\sigma_{\lambda,t}^2$ leads to a decline in the price-consumption ratio when θ is negative. Hence, under the preference configuration that ψ and γ are greater than 1, our model suggests that increased economic uncertainty, regardless of its nature, reduces the price-consumption ratio.

¹⁰ $\gamma > 1$ indicates that the representative agent prefers an early resolution of risks. In addition, θ is negative when ψ and γ are both greater than 1.

For equity assets, matching up the undetermined coefficients yields the solution to the A_d 's:

$$\begin{aligned}
A_{1,d} &= \frac{\phi - \frac{1}{\psi}}{1 - k_{1,d}\rho} , \\
A_{2,d} &= \frac{\rho_\lambda}{1 - k_{1,d}\rho_\lambda} , \\
A_{3,d} &= \frac{1}{2(1 - k_{1,d}v_1)} \left\{ \theta \left(1 - \frac{1}{\psi}\right)^2 + (\varphi_d)^2 + [(k_{1,d}A_{1,d} - k_{1,c}A_1)^2 + (k_{1,c}A_1)^2] (\varphi_e)^2 \right\} , \\
A_{4,d} &= \frac{1}{2(1 - k_{1,d}v_\lambda)} \left[\theta (1 + k_{1,c}A_2)^2 + (k_{1,d}A_{2,d} - k_{1,c}A_2)^2 \right] .
\end{aligned}$$

Once again, it turns out that the solution to the coefficients $A_{1,d}$ and $A_{3,d}$ coincides with its counterpart in the Bansal and Yaron (2004) model. As shown later, this property also applies to the solution to the risk-free rate. Therefore, by introducing time-preference shocks that are independent of the consumption and the cash flow processes, our model retains the long-run consumption growth channel exactly in the Bansal and Yaron (2004) fashion.

Exploring the solution forms of the A_d 's, we observe that positive time-preference shocks increase not only the price-consumption ratio, but also the price-dividend ratio. When $k_{1,d} > k_{1,c}$, the price-dividend ratio is more sensitive to changes in h_t than the price-consumption ratio. In addition, the price-dividend ratio rises in response to higher x_t if and only if $\phi > \frac{1}{\psi}$. In particular, when $k_{1,d} > k_{1,c}$ and $\phi > 1$, any realization of x_t will induce larger shifts in the price-dividend ratio than in the price-consumption ratio.¹¹ Lastly, the signs of $A_{3,d}$ and $A_{4,d}$ depend on the magnitude of the economic parameters, and in this case, are ambiguous. In section 2.3, we will infer the signs of $A_{3,d}$ and $A_{4,d}$ once we estimate the model parameters.

¹¹Note that this condition is sufficient but not necessary.

Plugging Equation (2.14) and (2.18) into (2.20), we solve for the stochastic discount factor and derive the difference between m_{t+1} and its conditional expectation:

$$\begin{aligned}
m_{t+1} - E_t(m_{t+1}) &= (\theta - 1) k_1 A_1 \varphi_e \sigma_t e_{t+1} + \left(\theta - \frac{\theta}{\psi} - 1 \right) \sigma_t \eta_{t+1} \\
&\quad [\theta + (\theta - 1) k_1 A_2] \sigma_{\lambda,t} \varepsilon_{t+1} + (\theta - 1) k_1 A_3 \sigma_w w_{t+1} \\
&\quad + (\theta - 1) k_1 A_4 \sigma_\pi \pi_{t+1} \\
&= -\beta_{m,e} \sigma_t e_{t+1} - \beta_{m,\eta} \sigma_t \eta_{t+1} - \beta_{m,\varepsilon} \sigma_{\lambda,t} \varepsilon_{t+1} \\
&\quad - \beta_{m,w} \sigma_w w_{t+1} - \beta_{m,\pi} \sigma_\pi \pi_{t+1} \quad .
\end{aligned} \tag{2.21}$$

Then, for any equity asset d , the expected equity premium is given by

$$\begin{aligned}
E_t(r_{d,t+1} - r_{f,t}) &= -cov_t(m_{t+1} - E_t(m_{t+1}), r_{d,t+1} - E_t(r_{d,t+1})) \\
&\quad - 0.5 var_t(r_{d,t+1}) \\
&= k_{1,d} A_{1,d} \varphi_e \beta_{m,e} \sigma_t^2 + k_{1,d} A_{2,d} \beta_{m,\varepsilon} \sigma_{\lambda,t}^2 \\
&\quad + k_{1,d} A_{3,d} \beta_{m,w} \sigma_w^2 + k_{1,d} A_{4,d} \beta_{m,\pi} \sigma_\pi^2 - 0.5 var_t(r_{d,t+1}) \\
&= \beta_{d,e} \beta_{m,e} \sigma_t^2 + \beta_{d,\varepsilon} \beta_{m,\varepsilon} \sigma_{\lambda,t}^2 + \beta_{d,w} \beta_{m,w} \sigma_w^2 \\
&\quad + \beta_{d,\pi} \beta_{m,\pi} \sigma_\pi^2 - 0.5 var_t(r_{d,t+1}) \quad ,
\end{aligned} \tag{2.22}$$

where $r_{f,t}$ denotes the risk-free rate; and $var_t(r_{d,t+1}) = (\beta_{d,e}^2 + \varphi_d^2) \sigma_t^2 + \beta_{d,\varepsilon}^2 \sigma_{\lambda,t}^2 + \beta_{d,w}^2 \sigma_w^2 + \beta_{d,\pi}^2 \sigma_\pi^2$. In equation (2.22), $\beta_{d,l}$ (where $l = e, \varepsilon, w$ and π) captures the exposure of the equity returns to the shock l , and $\beta_{m,l}$ represents the market price of the corresponding risks. For example, $\beta_{d,e}$ measures the extent to which $r_{d,t+1}$ is exposed to the uncertainty associated

with long-run consumption growth, and $\beta_{m,e}$ is the market price of the long-run risks. In addition, equation (2.22) suggests that the variation in the expected equity premium is purely driven by the volatilities of long-run consumption growth and time-preference shocks.

To close the model, we conjecture that $r_{f,t}$ is a linear combination of the state variables:

$$r_{f,t} = A_{0,rf} + A_{1,rf}x_t + A_{2,rf}h_t + A_{3,rf}\sigma_t^2 + A_{4,rf}\sigma_{\lambda,t}^2. \quad (2.23)$$

In Appendix B.1, we show that the coefficients of the A_{rf} 's are given by

$$\begin{aligned} A_{1,rf} &= \frac{1}{\psi}, \\ A_{2,rf} &= -\rho\lambda, \\ A_{3,rf} &= -\frac{1}{2} \left[\theta \left(1 - \frac{1}{\psi} \right)^2 + (\theta + 1) (k_{1,c}A_1\varphi_e)^2 + 1 \right], \\ A_{4,rf} &= -\frac{1}{2} \left[\theta (1 + k_{1,c}A_2)^2 + (k_{1,c}A_2)^2 \right]. \end{aligned}$$

First, the risk-free rate rises when long-run consumption growth becomes higher, and the magnitude of the risk-free rate adjustments to a given realization of x_t is exclusively determined by the IES parameter. Here, the agent's risks attitude does not matter. Second, positive time-preference shocks reduce the risk-free rate. Therefore, when the representative agent realizes that she would value more of future consumption than current consumption, the agent would tend to buy less of the risk-free asset and purchase more of the risky assets. Once again, however, the relationship between the risk-free rate and the economic uncertainties is undetermined. The signs of $A_{3,rf}$ and $A_{4,rf}$ are ambiguous, and thus, need to be inferred from the data.

2.3. Parameter Estimates

To assess the model's ability to account for the asset pricing facts, this study develops a two-step empirical approach that estimates the deep parameters and the latent state variables

from the medium-to-low frequency data. In the first step, we estimate the parameters that regulate the preference configuration and the exogenous processes using quarterly U.S. data ranging from 1948:Q1 to 2013:Q4. Our sample consists of five series. Those are, real per capita consumption growth, real dividend growth, real price-dividend ratio, real total market returns and real risk-free rate. Detailed description of data construction is provided in Appendix B.2.1.

Following Bansal, Kiku and Yaron (2012), this study estimates the vector of the economic parameters Θ , where

$$\Theta = \{\gamma, \psi, \delta, \rho, \rho_\lambda, \mu_c, \mu_d, \phi, \varphi_e, \varphi_d, \sigma_0, \sigma_\lambda, \bar{z}_c, v_1, v_\lambda, \sigma_w, \sigma_\pi\},$$

using GMM. In particular, we consider a two-step efficient GMM estimator that minimizes the distance between the model-implied population moments, $\Phi(\Theta)$, and the data-based sample moments Φ_d . Specifically, our GMM estimator $\hat{\Theta}$ is given by

$$\hat{\Theta} = \underset{\Theta}{\operatorname{argmin}} [\Phi(\Theta) - \Phi_d]' W(\Theta) [\Phi(\Theta) - \Phi_d], \quad (2.24)$$

where $W(\Theta)$ is the weighting matrix; and the variance-covariance matrix of the empirical moments is given by $[W(\Theta)]^{-1}$. In the GMM estimation procedure, we start from an identity weighting matrix in the first step, and then update $W(\Theta)$ in the second step using the Newey and West (1987) method to ensure that $\hat{\Theta}$ is Heteroscedasticity and Auto-Correlation Consistent (HAC).¹² In addition, we impose restrictions on the sizes of σ_w and σ_π to guarantee the positivity of σ_t^2 and $\sigma_{\lambda,t}^2$ (once we proceed to latent variable smoothing).¹³

¹²Following the rule of thumb, we compute the lag length as $1.2T^{(1/3)}$, where T denotes the sample size.

¹³To be specific, we impose the restriction that σ_w cannot exceed one tenth of the point estimate of σ_0 ; and σ_π cannot exceed one tenth of the point estimate of σ_λ

2.3.1. Moment Conditions with Time Aggregation

Our GMM estimation takes into account 22 moment conditions. In table 2.1, we categorize these moments into four sets. The first set of the moment conditions captures the key moments associated with the market fundamentals, namely the consumption growth and the dividend growth. Under the column “Asset Prices”, we include the first- and the second-moments of the price-dividend ratio, the risk-free rate, the market returns, and the equity premium. In particular, we incorporate the auto-covariance moment of the price-dividend ratio to make sure that the model will take into account the persistence of the price-dividend ratio dynamics. The moment conditions under the column “Predictability” emphasize the stylized fact that the price-dividend ratio is an important predictor of future market returns and future consumption growth. And the last set of the moment conditions focuses on the weak correlation between consumption growth and the market returns.¹⁴

It is worth mentioning that we perform time-aggregation on the observed variables based on their solution forms presented in section 2.2, and derive the analytical expressions of all the moment conditions listed in Table 2.1 accordingly. As pointed out by recent asset pricing studies, the conventional “temporal-aggregation” approach is unable to accurately extract information from the low-frequency data, and thus, tends to yield biased parameter estimates and distort model inference. For example, Bansal, Kiku and Yaron (2012) show that the parameter estimates of the long-run risk model using the temporal-aggregated moments are not only substantially distinct from their counterparts based on the time-aggregated moments, but also strongly rejected by the U.S. data.

Following the literature, this study assumes that the representative agent re-optimizes on the monthly basis.¹⁵ We let t and τ denote the time indices of the agent’s decision-making

¹⁴The stylized fact that consumption growth is weakly correlated with the market returns is also referred to as the weak correlation puzzle.

¹⁵Similar assumption can be found in Stambaugh (1991), Campbell and Cochrane (1999) and so forth. In addition, Bansal, Kiku and Yaron (2012) treat agents’ decision interval as an additional parameter to the long-run risk model. Their estimation result indicates that agents re-make their decisions approximately every 33 days.

Table 2.1. A List of Moment Conditions

Market Fundamentals	Asset Prices	Predictability	Weak Correlation
$E(\Delta c_\tau^Q)$	$E(z_{d,\tau}^Q)$	$cov(\Delta c_\tau^Q, z_{d,\tau-1}^Q)$	$cov(r_{m,\tau}^Q, \Delta c_\tau^Q)$
$var(\Delta c_\tau^Q)$	$var(z_{d,\tau}^Q)$	$cov(\Delta c_\tau^Q, z_{d,\tau-4}^Q)$	$cov(r_{m,\tau}^Q, \Delta c_{\tau-1}^Q)$
$cov(\Delta c_\tau^Q, c_{\tau-1}^Q)$	$cov(z_{d,\tau}^Q, z_{d,\tau-1}^Q)$	$cov(r_{m,\tau}^Q, z_{d,\tau-1}^Q)$	
$E(\Delta d_\tau^A)$	$E(r_{f,\tau}^Q)$	$cov(r_{m,\tau}^Q, z_{d,\tau-4}^Q)$	
$var(\Delta d_\tau^A)$	$var(r_{f,\tau}^Q)$		
$cov(\Delta d_\tau^A, d_{\tau-1}^A)$	$E(r_{m,\tau}^Q)$		
$cov(\Delta c_\tau^Q, d_\tau^A)$	$var(r_{m,\tau}^Q)$		
	$E(r_{m,\tau}^Q - r_{f,\tau}^Q)$		
	$var(r_{m,\tau}^Q - r_{f,\tau}^Q)$		

Note: First, τ denote the time index of the sampling frequency, and thus increments quarterly. Second, Q denotes that the moment conditions are time-aggregated over the quarter. Similarly, A implies that the moment conditions are time-aggregated at the annual frequency. Finally, all the moment conditions in Table 2.1 are unconditional moments.

interval and our sample frequency, respectively. Given that τ increments on the quarterly basis, the variable y_t (where $y = \Delta c, \Delta d, z_d, r_f$ and r_m) defined in section 2.2 is unobserved. To correct the information loss originating from the mismatch between t and τ , we need to pin down the relationship between y_t and the observable y_τ . For the return variables $r_{f,t}$ and $r_{m,t}$, it is straightforward to show that $r_{f,\tau}^Q$ and $r_{m,\tau}^Q$ satisfy

$$r_{f,\tau}^Q = \sum_{j=0}^2 r_{f,t-j}, \quad (2.25)$$

$$r_{m,\tau}^Q = \sum_{j=0}^2 r_{m,t-j}, \quad (2.26)$$

where Q indicates that the returns on the risk-free and the equity assets are measured on the quarterly basis. For consumption, we define the quarterly consumption growth Δc_τ^Q as

$$\Delta c_\tau^Q \equiv \log \left(\frac{\sum_{j=0}^2 C_{t-j}}{\sum_{j=0}^2 C_{t-3-j}} \right).$$

Using a first-order Taylor series expansion, we show (in Appendix B.3) that Δc_τ^Q can be approximated as

$$\Delta c_\tau^Q \approx \sum_{j=1}^3 \left(\frac{j}{3} \Delta c_{t+1-j} \right) + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) \Delta c_{t-2-j} \right]. \quad (2.27)$$

In terms of the dividend growth, we decide not to use the quarterly dividend growth Δd_τ^Q as the observed variable. This is primarily because the series of quarterly dividend growth in our sample exhibits relatively strong seasonality, and the seasonal pattern would implausibly introduce considerable fluctuations that are not accounted for by the model. Instead, we define Δd_τ^A as the percentage change in the dividend payment in quarter τ of year k relative to the dividend payment in quarter τ of year $k-1$. Then, we derive the

approximated relationship between Δd_τ^A and Δd_t :

$$\begin{aligned}
\Delta d_\tau^A &\equiv \log \left(\frac{D_\tau^Q}{D_{\tau-4}^Q} \right) \\
&= \log \left(\frac{\sum_{j=0}^2 D_{t-j}}{\sum_{j=0}^2 D_{t-12-j}} \right) \\
&\approx \sum_{j=1}^2 \left(\frac{j}{3} \Delta d_{t+1-j} \right) + \sum_{j=0}^9 (\Delta d_{t-2-j}) + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) \Delta d_{t-11-j} \right].
\end{aligned} \tag{2.28}$$

Lastly, the quarterly price-dividend ratio $\Delta z_{d,\tau}^Q$ is specified as the end-of-period stock price divided by the summation of the dividend payments over the quarter, which is given by

$$\Delta z_{d,\tau}^Q \equiv \log \left(\frac{P_{d,t}}{\sum_{j=0}^2 D_{t-j}} \right).$$

And a first-order Taylor series expansion yields

$$\Delta z_{d,\tau}^Q \approx z_{d,t} + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) \Delta d_{t+1-j} \right] - \log(3). \tag{2.29}$$

Given the exogenous processes and the equilibrium conditions, we rewrite y_τ as functions of the unobserved state variables and the innovations. This would allow us to derive the analytical expressions of all the moment conditions in Table 2.1. For example, rewriting equation (2.27) yields

$$\begin{aligned}
\Delta c_\tau^Q &\approx \sum_{j=1}^3 \left(\frac{j}{3} \Delta c_{t+1-j} \right) + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) \Delta c_{t-2-j} \right] \\
&= 3\mu + \left\{ \sum_{j=1}^3 \left(\frac{j}{3} x_{t-j} \right) + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) x_{t-3-j} \right] \right\} \\
&\quad + \left\{ \sum_{j=1}^3 \left(\frac{j}{3} \sigma_{t-j} \eta_{t+1-j} \right) + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) \sigma_{t-3-j} \eta_{t-2-j} \right] \right\}.
\end{aligned}$$

Hence, the unconditional mean of consumption growth, $E(\Delta c_\tau^Q)$, is given by

$$E(\Delta c_\tau^Q) = 3\mu,$$

and the unconditional variance $var(\Delta c_\tau^Q)$ satisfies¹⁶

$$var(\Delta c_\tau^Q) = \frac{1}{9(1-\rho^2)} (19 + 32\rho + 20\rho^2 + 8\rho^3 + 2\rho^4) (\varphi_e \sigma_0)^2 + \frac{19}{9} \sigma_0^2.$$

Derivations are presented in Appendix B.3.

2.3.2. Empirical Results

Table 2.2 displays our estimates of the deep parameters along with the standard errors. Under the baseline model¹⁷, the estimates of the risk aversion coefficient and the IES parameter are quite moderate ($\gamma = 1.7006$ and $\psi = 2.1908$).¹⁸ As shown in section 2.4, our model does not rely on a large risk aversion coefficient to resolve the equity premium puzzle. The point estimate of the subjective discount factor is 0.9999, a value that is slightly higher than those reported in the literature. However, the δ estimate is not necessarily unreasonable, because our model allows the actual subjective discount factor to deviate from its steady state (or long-run value) when there are changes in time-preferences.

¹⁶Note that the temporal-aggregation approach would yield the following analytical expressions of the first- and second-moments of quarterly consumption growth:

$$E(\Delta c_\tau^Q) = \mu;$$

$$var(\Delta c_\tau^Q) = \left[\frac{1}{(1-\rho^2)} + 1 \right] \sigma_0^2.$$

¹⁷The baseline specification refers to the model that accommodates both the long-run consumption growth and the time-preference shocks channels.

¹⁸Several studies, such as Hall (1988), Campbell (2003), and Beeler and Campbell (2012), suggest that the parameter of IES is likely to be less than 1. In particular, Beeler and Campbell (2012) argue that the IES estimates based upon standard long-run risks models are not supported by the consumption and the risk-free rate data. This study, however, justifies the IES estimates in the long-run risk literature and shows that the presence of time-preference shocks weakens the tendency for consumption growth to move predictably with the short-term real interest rate. Further discussion is provided in section 2.4.

Table 2.2. Parameter Estimates

Parameters	Baesline		LRC Only		TPS Only	
	Estimate	SE	Estimate	SE	Estimate	SE
γ	1.7006	3.3935	1.4027	3.0633	1.2136	12.8738
ψ	2.1908	0.1028	4.3143	1.3480	3.0248	13.6290
δ	0.9999	0.0254	0.9943	0.0183	0.9999	0.1119
ρ	0.9718	$5.25e^{-4}$	0.9954	$5.98e^{-4}$	/	/
ρ_λ	0.1700	1.0329	/	/	0.1760	0.0858
μ	0.0016	$1.28e^{-4}$	0.0014	$1.34e^{-4}$	0.0020	$1.20e^{-4}$
μ_d	0.0049	$3.78e^{-4}$	0.0015	$1.79e^{-4}$	0.0024	$2.35e^{-4}$
ϕ	2.8266	0.0144	1.6199	0.0238	/	/
φ_e	0.1042	$9.58e^{-4}$	0.0027	$1.76e^{-4}$	/	/
φ_d	1.2094	0.0294	$2.83e^{-5}$	4.2882	2.6234	0.0127
σ_0	0.0133	$4.60e^{-4}$	0.1223	0.0032	0.0136	$2.13e^{-4}$
σ_λ	0.0803	0.5563	/	/	0.0873	0.0533
\bar{z}_c	4.5774	107.18	5.0385	1.1552	2.4859	41.6963
v_1	0.8531	217.94	0.5354	0.0022	0.9556	155.1754
v_λ	0.3446	3.7331	/	/	0.9999	0.0891
σ_w	$8.83e^{-6}$	0.0053	$1.28e^{-5}$	4.6001	$9.26e^{-6}$	0.0132
σ_π	$1.30e^{-5}$	10.6396	/	/	$9.94e^{-6}$	0.0289
χ^2 -test	31.4967		15.6013		27.5533	
p -value	$7.47e^{-6}$		0.0757		0.0006	

For the parameters that govern the exogenous processes, several findings are worth mentioning. First, the process of the long-run consumption growth is highly persistent. The parameter value of ρ ($\rho = 0.9718$) is not only consistent with the definition of x_t , but also close to the estimates reported in existing long-run risks studies. In contrast, the point estimate of the auto-regression coefficient ρ_λ is only 0.17, which is considerably different from the estimate in Albuquerque, Eichenbaum and Rebelo (2013).¹⁹ Second, the estimate of φ_e is 0.1042, indicating that the estimated volatility of long-run consumption growth is only one-tenth of the volatility of aggregate consumption growth. In addition, the point estimates of ϕ and φ_d are both greater than 1. Hence, our model predicts that dividend growth fluctuates more than consumption growth in response to shocks to long-run consumption growth and long-run risks. Furthermore, the process of the long-run consumption growth volatility is relatively persistent ($v_1 = 0.8531$), but its variation is quantitatively small ($\sigma_w = 8.83e^{-6}$). The point estimate of the σ_π also suggests that there is very little variation in the volatility of time-preference shocks. Moreover, the implied values of $A_{3,d}$, $A_{3,rf}$, $A_{4,d}$ and $A_{4,rf}$, are 121.8241, 0.0999, -30.1288 and -3.7411, respectively. These findings indicate that the responses of asset market variables to fluctuating economic uncertainties will depend on the nature of the risks. Under our framework, risk-free rate and price-dividend ratio rise in response to increased uncertainty in long-run consumption growth, and fall in response to higher valuation risks.

In Table 2.2, we also report the parameter estimates under two alternative model specifications. Here, “LRC Only” refers to the model that consists of exclusively the long-run consumption growth channel; and analogously, “TPS Only” refers to the model that only incorporates the time-preference shocks channel. Note that, excluding the time-preference shocks channel, our model reduces to the standard long-run risks model as in Bansal and Yaron (2004). Omitting the long-run consumption growth component, however, the model

¹⁹Albuquerque, Eichenbaum and Rebelo (2013) specify the time-preference shocks as a highly persistent process. Their point estimate of the auto-regression coefficient is 0.995. In this study, we provide empirical evidence showing that the process of time-preference shocks is unlikely to be persistent. Further discussion is provided in section 2.4.

is not equivalent to that proposed in Albuquerque, Eichenbaum and Rebelo (2013). This is because the “TPS Only” specification still allows the volatilities of consumption growth, dividend growth and time-preference shocks to vary over time. On the one hand, the point estimates of the risk aversion coefficient and the auto-regression coefficients (ρ and ρ_λ) using the two alternatives are found to be close to their counterparts in the baseline model; and the estimates of σ_w and σ_π further confirm that the volatilities of σ_t and $\sigma_{\lambda,t}$ are quantitatively small. On the other hand, several parameter estimates using these alternative specifications are noticeably different. First, both the “LRC Only” and the “TPS Only” models return the IES estimates that further depart from the generally agreed value in the literature. Second, the unconditional mean of long-run consumption growth volatility under the “LRC Only” model ($\sigma_0 = 0.1223$) is almost ten times larger than the point estimate in the baseline model. In addition, v_λ estimate in the “TPS Only” model is 0.9999, indicating a highly persistent process of $\sigma_{\lambda,t}^2$.

Given the parameter estimates, this section follows the conventional wisdom by evaluating the models’ ability to match the key moments of the market fundamentals and the asset market variables. For each specification, we simulate the model for 150 periods and repeat for 10,000 times. The model-implied moments along with the t -statistics are summarized in Table 2.3. Even though the model-implied standard deviation and the auto-correlation of dividend growth under the baseline and the “TPS Only” specifications are slightly higher than the sample moments, all three models match the moments related to the market fundamentals fairly well. However, the “TPS Only” model seems unable to match the standard deviation and the auto-correlation of the price-dividend ratio. This finding implies that the predictable long-run consumption growth is likely to be an influential component that induces the variation in the price-dividend ratio. In addition, the simulated moments of $corr(r_{m,\tau}^Q, \Delta c_{\tau-1}^Q)$ across all model specifications suggest that excluding the time-preference shocks does not weaken the model’s ability to replicate the weak correlation between market returns and consumption growth. Nevertheless, we find that time-preference shocks might be potentially important to understanding the movements in the return variables. As shown in

Table 2.3. Model-Implied Moments

Moments	Baseline			LRC Only		TPS Only	
	Sample Moments	Simulated Moments	t -Stats Difference	Simulated Moments	t -Stats Difference	Simulated Moments	t -Stats Difference
$E(\Delta c_\tau^Q)$	0.0051	0.0059	0.0631	0.0049	-0.0087	0.0059	0.3350
$std(\Delta c_\tau^Q)$	0.0056	0.0668	8.9897	0.1265	9.8638	0.0141	6.2442
$corr(\Delta c_\tau^Q, c_{\tau-1}^Q)$	0.2416	0.2172	-0.1804	0.1744	-0.4925	0.1723	-0.5117
$E(\Delta d_\tau^A)$	0.0060	0.0293	0.3375	0.0212	0.5636	0.0293	0.8983
$std(\Delta d_\tau^A)$	0.0252	0.1809	4.1416	0.0175	-1.1320	0.0810	3.8968
$corr(\Delta d_\tau^A, d_{\tau-1}^A)$	0.4175	0.8608	11.3295	0.9479	24.5878	0.7559	4.9543
$corr(\Delta c_\tau^Q, d_\tau^A)$	0.0956	0.2012	0.6500	0.0198	-0.4309	-0.0019	-0.5800
$E(z_{d,\tau}^Q)$	4.8858	4.8919	0.0895	4.8964	0.0634	4.9534	21.5442
$std(z_{d,\tau}^Q)$	0.4085	0.2238	-5.7383	0.1132	-7.2161	0.0192	-260.71
$corr(z_{d,\tau}^Q, z_{d,\tau-1}^Q)$	0.9766	0.5265	-3.6020	0.8533	-1.4754	-0.0168	-6.9413
$E(r_{f,\tau}^Q)$	0.0031	-0.0049	-0.2798	0.0247	21.7002	-0.0055	-156.87
$std(r_{f,\tau}^Q)$	0.0064	0.0849	5.7439	$6.64e^{-4}$	-23.4449	$3.25e^{-5}$	-475.85
$E(r_{m,\tau}^Q)$	0.0182	0.0046	-0.7522	0.0028	-1.9674	0.0037	-2.2418
$std(r_{m,\tau}^Q)$	0.0838	0.2068	5.9486	0.0474	-8.1707	0.0443	-9.6919
$corr(r_{m,\tau}^Q, \Delta c_\tau^Q)$	0.2070	0.0375	-1.1926	-0.0044	-1.4674	$1.33e^{-4}$	-1.4417
$corr(r_{m,\tau}^Q, \Delta c_{\tau-1}^Q)$	-0.0279	$6.17e^{-4}$	0.1904	-0.0037	0.1644	0.0016	0.1996
$corr(\Delta c_\tau^Q, z_{d,\tau-1}^Q)$	-0.0085	0.1404	0.9132	0.0208	0.1709	-0.0012	0.0490
$corr(\Delta c_\tau^Q, z_{d,\tau-4}^Q)$	-0.0986	0.0046	0.5954	0.0149	0.6400	$-9.78e^{-4}$	0.6413
$corr(r_{m,\tau}^Q, z_{d,\tau-1}^Q)$	-0.1335	-0.3581	-2.2555	-0.1905	-0.4714	-0.0189	0.7954
$corr(r_{m,\tau}^Q, z_{d,\tau-4}^Q)$	-0.1119	-0.0881	0.1780	-0.1525	-0.3240	-0.0165	0.6330
$E(r_{m,\tau}^Q - r_{f,\tau}^Q)$	0.0151	0.0095	-0.1794	-0.0219	-5.2834	0.0092	-0.9119
$std(r_{m,\tau}^Q - r_{f,\tau}^Q)$	0.0842	0.2302	6.5722	0.0473	-8.3019	0.0443	-9.7977

Table 2.3, the baseline model makes the most accurate prediction of the moments associated with the return variables. While the baseline specification moderately over-predicts the standard deviation of the market returns and the risk-free rate, the two competing alternatives cannot replicate the key moments of the returns and the equity premium.²⁰ The difficulty is especially pronounced under the “LRC Only” specification, because the simulated mean of the equity premium is negative and significantly rejected by the data. To rule out the possibility that such failure is caused by our GMM estimator, we simulate the “LRC Only” model using the parameter estimates reported in Bansal, Kiku and Yaron (2012). Table 2.4 summarizes the statistics. Unfortunately, we find that using the Bansal, Kiku and Yaron (2012) calibration does not improve the model’s ability to match the auto-correlation of the price-dividend ratio, the standard deviation of the risk-free rate, or the unconditional mean of the equity premium.

From our perspective, these pieces of empirical evidence point to the possibility that both of the long-run consumption growth and the time-preference shocks channels are of central importance to explaining the asset price movements over the business cycles. Unfortunately, moment-based approaches seem unable to provide sufficient information to confirm our conjecture. It is primarily because modern asset pricing models that take into account a large set of pre-selected moment conditions would unavoidably fail along several dimensions. And the extent to which those mismatched (as well as the omitted) moments weaken the model’s credibility seems not answered by the moment-based test statistics. In addition, moment-based methods do not yield any direct implications on the relative importance of the investigated macroeconomic sources of asset market fluctuations. Given these concerns, we proceed by extracting the latent state variables from the data and assessing the model’s ability to fit the joint dynamics of the price-dividend ratio and the return variables. Further discussion is provided in section 2.4.

²⁰Based on several experiments on the model parameters, we find that models that incorporate time-preference shocks are likely to underpredict the standard deviation and the auto-correlation coefficient of the price-dividend ratio. Therefore, under the baseline model, the parameter choices that match these two moments relatively well tend to generate higher standard deviation of the return variables and the equity premium.

Table 2.4. Model-Implied Moments under “LRC Only” Specification with BKY Calibration

Parameter	Calibration	Moments	Sample Moments	Simulated Moments	t -Stats Difference
γ	7.43	$E\left(\Delta c_\tau^Q\right)$	0.0051	0.0037	-0.6385
ψ	2.05	$std\left(\Delta c_\tau^Q\right)$	0.0056	0.0077	2.7448
δ	0.9989	$corr\left(\Delta c_\tau^Q, c_{\tau-1}^Q\right)$	0.2416	0.2044	-0.2654
ρ	0.9812	$E\left(\Delta d_\tau^A\right)$	0.0060	0.0261	0.5251
ρ_λ	/	$std\left(\Delta d_\tau^A\right)$	0.0252	0.0872	3.8283
μ	0.0012	$corr\left(\Delta d_\tau^A, d_{\tau-1}^A\right)$	0.4175	0.7729	5.3254
μ_d	0.0020	$corr\left(\Delta c_\tau^Q, d_\tau^A\right)$	0.0956	0.0571	-0.2214
ϕ	4.4500	$E\left(z_{d,\tau}^Q\right)$	4.8858	4.9904	1.1803
φ_e	0.0306	$std\left(z_{d,\tau}^Q\right)$	0.4085	0.0895	-12.1456
φ_d	5.0000	$corr\left(z_{d,\tau}^Q, z_{d,\tau-1}^Q\right)$	0.9766	0.7747	-1.7968
σ_0	0.0073	$E\left(r_{f,\tau}^Q\right)$	0.0031	0.0054	2.9378
σ_λ	/	$std\left(r_{f,\tau}^Q\right)$	0.0064	$7.84e^{-4}$	-22.9284
\bar{z}_c	3.0000	$E\left(r_{m,\tau}^Q\right)$	0.0182	0.0018	-1.7035
v_1	0.9983	$std\left(r_{m,\tau}^Q\right)$	0.0838	0.0619	-3.5140
v_λ	/	$corr\left(r_{m,\tau}^Q, \Delta c_\tau^Q\right)$	0.2070	-0.0060	-1.4725
σ_w	$2.62e^{-6}$	$corr\left(r_{m,\tau}^Q, \Delta c_{\tau-1}^Q\right)$	-0.0279	-0.0133	0.1000
σ_π	/	$corr\left(\Delta c_\tau^Q, z_{d,\tau-1}^Q\right)$	-0.0085	0.1839	1.1136
		$cor\left(\Delta c_\tau^Q, z_{d,\tau-4}^Q\right)$	-0.0986	0.1162	1.1638
		$corr\left(r_{m,\tau}^Q, z_{d,\tau-1}^Q\right)$	-0.1335	-0.0999	0.2426
		$corr\left(r_{m,\tau}^Q, z_{d,\tau-4}^Q\right)$	-0.1119	-0.0814	0.2146
		$E\left(r_{m,\tau}^Q - r_{f,\tau}^Q\right)$	0.0151	-0.0037	-2.0464
		$std\left(r_{m,\tau}^Q - r_{f,\tau}^Q\right)$	0.0842	0.0618	-3.6097

Note: Calibration follows the parameter estimates in Bansal, Kiku and Yaron (2012) except for \bar{z}_c , since the estimate of \bar{z}_c is not reported in their study. According to our experiments, the choice of \bar{z}_c does not noticeably affect the simulation results.

2.4. Latent Variable Smoothing

As mentioned earlier, the existing long-run risks literature devotes quite limited effort to replicating the dynamic paths of the asset market variables. A major difficulty that complicates this fitting task lies in coming up with a reasonable empirical approach that is capable of extracting the unobserved state variables from the low-frequency data. Conventional long-run risks studies, such as Bansal, Kiku and Yaron (2007), employ constrained linear regressions to recover the unobserved long-run consumption growth component and its conditional volatility from the consumption and the risk-free rate data. Bansal, Kiku and Yaron (2012) refine the estimation methodology via performing grid-searches over a two-dimensional space, in which the latent state variables are jointly estimated with the GMM estimator through minimizing a weighted quadratic loss function of the price-dividend ratio and the risk-free rate. Nevertheless, two major problems are associated with these approaches. First, instead of using the asset market variables (especially the market returns) which seem to contain considerable information about the unobserved driving forces, the aforementioned estimation procedures rely on the data on risk-free rate and consumption growth. Given that the risk-free rate fluctuates too little (relative to other asset market variables) and consumption growth data contains too much noise that is difficult to filter out, it is questionable whether these variables would help to identify the long-run consumption growth component, which, by definition, is small in magnitude. More importantly, the latent state estimates reported in Bansal, Kiku and Yaron (2007, 2012) are not particularly convincing, because their approaches overlook the non-linear structure of the exogenous processes, and thus seem unable to accurately handle the non-linear tracking problems.

In this study, we infer the latent states from the monthly U.S. data on consumption growth, dividend growth, price-dividend ratio, risk-free rate and market returns. So as to overcome the difficulties induced by the non-linear structure of the exogenous processes, we propose a Sequential Monte Carlo approach that approximates the distribution of the state variables using the particle filter and the particle smoother. Exploiting the particle filter algorithm, Schorfheide, Song and Yaron (2014) propose a mixed-frequency Bayesian

approach to estimate the long-run risks model. While the spirit of their study is largely similar to ours, there are a few key differences. First, Schorfheide, Song and Yaron (2014) simultaneously estimate the economic parameters and the state variables using Bayesian method, whereas our study consists of two steps, a parameter estimation step followed by a latent variable smoothing step.²¹ Second, Schorfheide, Song and Yaron (2014) employ a mixed-frequency approach that uses both annual (prior to 1959) and monthly (after 1959) consumption data. This complexity further leads to a state vector that is inflated with over twenty variables. In contrast, we estimate the model parameters using consumption and financial data at the quarterly frequency, and then construct another data set at the monthly frequency to perform latent variable smoothing. As a result, our filtering and smoothing algorithms only rest on a state vector of four variables, and hence, are of more computational efficiency. In addition, Schorfheide, Song and Yaron (2014) assume that the variance of the measurement errors is homoscedastic. Our method, however, computes the standard deviation of the measurement errors in each period.

2.4.1. Particle Filtering and Particle Smoothing Algorithms

To perform latent variable smoothing, first, we rewrite the model in a state-space representation:

$$S_t = f(S_{t-1}, W_t; \Theta) , \quad (2.30)$$

$$Y_t = g(S_t, V_t; \Theta) , \quad (2.31)$$

where S_t denotes the vector of the latent state variables; Y_t represents the vector of the observables; W_t is a vector of the exogenous shocks; and V_t denotes the vector of error terms that are assumed to be *i.i.d.* standard normal and mutually independent. The transition

²¹Note that our parameter estimates (in section 2.3) are based on GMM, and thus do not rely on the approximation of the likelihood function. In the second step, latent state variables are smoothed taking as given the parameter estimates provided in section 2.3.

equation is regulated by our assumptions on the exogenous processes, and thus satisfies

$$S_t = C + FS_{t-1} + Q_{t-1}W_t, \quad (2.32)$$

where

$$S_t = [x_t, h_t, \sigma_t^2, \sigma_{\lambda,t}^2]' ;$$

$$W_t = [e_t, \varepsilon_t, w_t, \pi_t]' ;$$

$$C = [0, 0, (1 - v_1)\sigma_0^2, (1 - v_\lambda)\sigma_\lambda^2]' ;$$

$$F = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & \rho_\lambda & 0 & 0 \\ 0 & 0 & v_1 & 0 \\ 0 & 0 & 0 & v_\lambda \end{bmatrix} ;$$

and

$$Q_{t-1} = \begin{bmatrix} \varphi_e \sigma_{t-1} & 0 & 0 & 0 \\ 0 & \sigma_{\lambda,t-1} & 0 & 0 \\ 0 & 0 & \sigma_w & 0 \\ 0 & 0 & 0 & \sigma_\pi \end{bmatrix} .$$

Based on the solution to the equilibrium conditions, we construct the measurement equation as

$$Y_t = B + H_1 S_t + H_2 S_{t-1} + M_t V_t, \quad (2.33)$$

where

$$Y_t = [\Delta c_t, z_{d,t}, r_{f,t}, (r_{d,t} - g_{d,t})]' ;$$

$$V_t = [\eta_t, \vartheta_{1,t}, \vartheta_{2,t}, \vartheta_{3,t}]' ;$$

$$B = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} = \begin{bmatrix} \mu \\ A_{0,d} \\ A_{0,rf} \\ k_{0,d} + (k_{1,d} - 1)A_{0,d} \end{bmatrix} ;$$

$$H_1 = \begin{bmatrix} H_{1,1} \\ H_{1,2} \\ H_{1,3} \\ H_{1,4} \end{bmatrix} ;$$

$$= \begin{matrix} & & & \\ & & & \\ 3 \times 4 & \begin{bmatrix} 0 & 0 & 0 & 0 \\ A_{1,d} & A_{2,d} & A_{3,d} & A_{4,d} \\ A_{1,rf} & A_{2,rf} & A_{3,rf} & A_{4,rf} \\ k_{1,d}A_{1,d} & k_{1,d}A_{2,d} & k_{1,d}A_{3,d} & k_{1,d}A_{4,d} \end{bmatrix} \end{matrix}$$

$$H_2 = \begin{bmatrix} H_{2,1} \\ H_{2,2} \\ H_{2,3} \\ H_{2,4} \end{bmatrix} ;$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -A_{1,d} & -A_{2,d} & -A_{3,d} & -A_{4,d} \end{bmatrix}$$

and

$$M_t = \begin{bmatrix} \sigma_{t-1} & 0 & 0 & 0 \\ 0 & m_{1,t} & 0 & 0 \\ 0 & 0 & m_{2,t} & 0 \\ 0 & 0 & 0 & m_{3,t} \end{bmatrix} .$$

In particular, $\vartheta_{i,t}$ denotes the measurement error with standard deviation $m_{i,t}$ for $i = 1, 2$ and 3.

Given the state-space representation, we estimate the filtering density $p(S_t|Y_t; \Theta)$ using the Sequential Importance Resampling (SIR) particle filter. The idea of the SIR particle filter is to form the empirical distribution of S_t conditional on Y_t from mass points (or particles):

$$p(S_t|Y_t; \Theta) \simeq \sum_{j=0}^N q_t^j \delta_{S_t^i}(S_t), \quad \sum_{j=0}^N q_t^j = 1, \quad q_t^i \geq 0. \quad (2.34)$$

In equation (2.34), q_t^i is the weight attached to the i -th particle S_t^i , and δ denotes the Dirac Delta function. In each period, the SIR particle filter consists of two major steps. First, we

use the law of motion of the state variables to compute the conditional density $p(S_t|Y_{t-1}; \Theta)$ for N particles. Then, in the importance resampling step, we reweight the proposed density to avoid degeneracy, and further update the conditional density from $p(S_t|Y_{t-1}; \Theta)$ to $p(S_t|Y_t; \Theta)$. A detailed description of the algorithm is as follows:²²

Algorithm 1. Particle Filtering

Step 1: Set the parameter vector Θ , and the initial state vector $S_{0|0}$.

Step 2: At $t = 1$, use equation (2.32) and $\{S_{t-1|t-1}^i\}_{i=1}^N$ to draw $\{S_{t|t-1}^i\}_{i=1}^N$.

Step 3: Given $\{S_{t-1|t-1}^i\}_{i=1}^N$, $\{S_{t|t-1}^i\}_{i=1}^N$ and equation (2.33), compute $V_t^i = [\eta_t^i, \vartheta_{1,t}^i, \vartheta_{2,t}^i, \vartheta_{3,t}^i]'$ for $i = 1, 2, \dots, N$, where

$$\eta_t^i = \frac{1}{\sigma_{t-1}^i} [\Delta c_t - (B_1 + H_{1,1}S_{t|t-1}^i + H_{2,1}S_{t-1|t-1}^i)] ,$$

$$\vartheta_{1,t}^i = \frac{1}{m_{1,t}} [z_{d,t} - (B_2 + H_{1,1}S_{t|t-1}^i + H_{2,1}S_{t-1|t-1}^i)] ,$$

$$\vartheta_{2,t}^i = \frac{1}{m_{2,t}} [r_{f,t} - (B_3 + H_{1,2}S_{t|t-1}^i + H_{2,2}S_{t-1|t-1}^i)] ,$$

$$\vartheta_{3,t}^i = \frac{1}{m_{3,t}} [(r_{d,t} - g_{d,t}) - (B_4 + H_{1,3}S_{t|t-1}^i + H_{2,3}S_{t-1|t-1}^i)] .$$

In particular, the standard deviation of the measurement errors $m_{j,t}$ for $j = 1, 2, 3$ is computed over N particles.

Step 4: Calculate $p(Y_t|W^{t,i}, S_{t|t-1}^i, S_{t-1|t-1}^i; \Theta)$, which is given by

$$p(Y_t|W^{t,i}, S_{t|t-1}^i, S_{t-1|t-1}^i; \Theta) = p(V_t^i) \cdot |dY(V_t^i)|^{-1} ,$$

where $|dY(V_t^i)|$ is the absolute value of the determinant of the Jacobian of Y_t w.r.t. V_t^i . To be specific, for $i = 1, 2, \dots, N$,

$$p(V_t^i) = (2\pi)^{-\frac{4}{2}} \exp\left(-\frac{V_t^i V_t^{i'}}{2}\right) ,$$

²²The latent variable smoothing method in this study closely follows Fernndez-Villaverde and Rubio-Ramrez (2007), Godsill, Doucet and West (2012), and Born and Pfeifer (2014).

$$|dY(V_t^i)| = (\sigma_{t-1}^i m_{1,t} m_{2,t} m_{3,t}) .$$

Step 5: For each particle i at time t , compute the associated weight q_t^i , which is given by

$$q_t^i = \frac{p(Y_t|W^{t,i}, S_{t|t-1}^i, S_{t-1|t-1}^i; \Theta)}{\sum_{j=1}^N p(Y_t|W^{t,j}, S_{t|t-1}^j, S_{t-1|t-1}^j; \Theta)} ;$$

then perform systematic resampling by choosing $S_{t|t} = S_{t|t-1}^i$ with probability q_t^i , and draw the sequence of $\left\{S_{t|t}^i\right\}_{i=1}^N$.²³

Step 6: Repeat Step 2 - 5 for $t = 2, 3, \dots, T$, and terminate at $t = T$.

In addition to drawing the sequence of $\left\{S_{t|t}^i\right\}_{i=1}^N$ from the filtering density $p(S_t|Y_t; \Theta)$, this study also considers to infer the latent variables using the particle smoother, a backward-smoothing routine that approximates the historical distribution of the states conditional on the entire sample. Given the transition equation and the Markovian feature of the model, the smoothing density $p(S_t|S_{t+1:T}, Y_T; \Theta)$ satisfies

$$\begin{aligned} p(S_t|S_{t+1:T}, Y_T; \Theta) &= p(S_t|S_{t+1}, Y_t; \Theta) \\ &= \frac{p(S_t|Y_t; \Theta)f(S_{t+1}|S_t)}{p(S_{t+1}|Y_t; \Theta)} \\ &\propto p(S_t|Y_t; \Theta) f(S_{t+1}|S_t) , \end{aligned}$$

where $f(S_{t+1}|S_t)$ denotes the state evolution density. Accordingly, the historical distribution of S_t can be approximated by

$$p(S_t|S_{t+1}, Y_t; \Theta) \simeq \sum_{j=0}^N q_{t|t+1}^j \delta_{S_t^j}(S_t) . \quad (2.35)$$

²³For detailed discussion on systematic resampling, please refer to Arulampalam, Maskell, Gordon and Clapp (2002).

In equation (2.35), $q_{t|t+1}^i$ denotes the modified weight associated with particle i in period t conditional on the smoothed states in period $t + 1$, which is given by

$$q_{t|t+1}^i = \frac{q_t^i f(S_{t+1}|S_t^i)}{\sum_{j=1}^N q_t^j f(S_{t+1}|S_t^j)}, \quad (2.36)$$

where q_t^i is the weight computed from the particle filtering algorithm. To distinguish between the draws from the filtering and the smoothing densities, we let $\tilde{S}_t^i = [\tilde{x}_t^i, \tilde{h}_t^i, (\tilde{\sigma}_t^i)^2, (\tilde{\sigma}_{\lambda,t}^i)^2]$ denote the vector of the smoothed states, and summarize the smoothing algorithm as follows:

Algorithm 2. Particle Smoothing

Step 1: Set the parameter vector Θ , and draw the sequence of $\{\tilde{S}_T^i\}_{i=1}^N$ by choosing $\tilde{S}_T = S_{T|T}^i$ with probability q_T^i .

Step 2: Set $t = T - 1$; based on Equation (2.32), use $\{S_{t|t}^i\}_{i=1}^N$ and $\{\tilde{S}_{t+1}^i\}_{i=1}^N$ to compute $W_{t+1}^i = [e_{t+1}^i, \varepsilon_{t+1}^i, w_{t+1}^i, \pi_{t+1}^i]'$ for each particle i , where

$$e_{t+1}^i = \frac{1}{\sigma_{t|t}^i} [\tilde{x}_{t+1}^i - \rho x_{t|t}^i],$$

$$\varepsilon_{t+1}^i = \frac{1}{\sigma_{\lambda,t|t}^i} [\tilde{h}_{t+1}^i - \rho_\lambda h_{t|t}^i],$$

$$w_{t+1}^i = \frac{1}{\sigma_w} \left\{ [(\tilde{\sigma}_{t+1}^i)^2 - \sigma_0^2] - v_1 [(\sigma_{t|t}^i)^2 - \sigma_0^2] \right\},$$

$$\pi_{t+1}^i = \frac{1}{\sigma_w} \left\{ [(\tilde{\sigma}_{\lambda,t+1}^i)^2 - \sigma_\lambda^2] - v_\lambda [(\sigma_{\lambda,t|t}^i)^2 - \sigma_\lambda^2] \right\}.$$

Then, Calculate $f(\tilde{S}_{t+1}^i|S_{t|t}^i)$, which is given by

$$f(\tilde{S}_{t+1}^i|S_{t|t}^i) = (2\pi)^{-\frac{4}{2}} \exp\left(-\frac{W_{t+1}^i W_{t+1}^{i'}}{2}\right).$$

Step 3: Based on Equation (2.36), use $\{f(\tilde{S}_{t+1}^i|S_{t|t}^i)\}_{i=1}^N$ and $\{q_t^i\}_{i=1}^N$ to compute the modified weight $q_{t|t+1}^i$ for each particle i ; then resample by choosing $\tilde{S}_t = S_{t|t}^i$ with probability $q_{t|t+1}^i$, and draw the sequence of $\{\tilde{S}_t^i\}_{i=1}^N$.

Step 4: Repeat Step 2 - 3 in a backward fashion for $t = T - 2, T - 3, \dots, 1$, and terminate at $t = 1$.

2.4.2. Empirical Findings

Exploiting the aforementioned algorithms, we estimate the latent states (namely x_t, h_t, σ_t^2 and $\sigma_{\lambda,t}^2$) using 5,000 particles, and plot the historical distribution along with the mean and the median in Panel 1 - 4 of Figure 2.1.²⁴ We find that the mean of the smoothed long-run consumption growth x_t is mostly negative before 1990s, and rises rapidly during 1990s. After reaching its peak (at around 1.5%) in the year 2000, long-run consumption growth gradually falls to 0.5% from 2000 to 2007, and decreases further below zero during the Great Recession periods. An important feature shown in Panel 1 is that recessions are always associated with quick and substantial declines in long-run consumption growth. This pattern is especially pronounced during the 1973-1975 and the 2008-2009 recession periods. In contrast, time-preference shocks do not fluctuate as much as does long-run consumption growth. Panel 2 suggests that the mean of the smoothed h_t only deviate from the steady state within a small interval throughout the estimation periods, implying that agents' time-preferences are relatively stable across time. In addition, we find that agents do not immediately become impatient when confronting business recessions or financial crisis. Instead of having a strong willingness to consume more resources at the beginning of the recession, agents tend to assign more weight on future consumption. As the recession goes on and lasts for long enough periods, agents, at certain turning point, suddenly become impatient, and then tend to consume more in current than in future periods. Moreover, the smoothed volatility processes reported in Panel 3 and 4 show that the economic measure of uncertainties associated with long-run consumption growth and time-preferences are quantitatively small. However, the dynamic patterns of these two sources of risks are noticeably different when the economy is

²⁴We restrict our attention to the estimates of the historical distribution of the state variables using the particle smoothing algorithm. As shown in Figure B.1 in Appendix B.4, the filtered states are not noticeably different. In addition, we find that latent variable smoothing using 1,000 particles delivers largely similar results.

in the recession state. The smoothed long-run risks σ_t^2 usually rises during recessions, the feature of which is particularly highlighted in the Great Recession periods. To the contrary, the smoothed dynamics of valuation risks $\sigma_{\lambda,t}^2$ does not exhibit any systematic or perceivable changes in the recession states.

Based upon the smoothed state variables, we evaluate the model's ability to replicate the observables in the Y_t vector. Panel 1 - 4 in Figure 2.2 compare the model-implied dynamics of the observables with the actual data. In general, the baseline model is capable of explaining the joint dynamics of the price-dividend ratio, the risk-free rate, the market returns and the equity premium. First, the model-implied price-dividend ratio is fairly close to the actual price-dividend ratio throughout the sample. In particular, the model is able to account for the long swings of the price-dividend ratio dynamics. Second, for the risk-free rate, the dynamic path implied by the model is quite similar to that of the actual data prior to 1991. After 1991, while generating the risk-free rate that is systematically higher than the actual risk-free rate, the model can largely replicate the key qualitative features of the data. In addition, the market returns and the equity premium are also fitted well, even though the variation in the model-implied series is slightly lower than that in the data. Note that our model attributes the differences between the smoothed series and the actual data to the measurement errors. Overall, the measurement errors associated with the price-dividend ratio and the risk-free are small over the entire sample. For the market returns, the measurement errors are relatively sizable especially when certain extreme event happens (such as the stock market crashes in early 1987, late 1998 and the Great Recession periods). A potential explanation to this phenomenon is that our endowment economy setting does not capture the contagious effect of the global financial markets on the domestic market. Even when the state of the domestic economy remains unchanged, domestic market returns might respond to the failure in foreign stock markets. Therefore, it is not surprising to see that our framework, which does not explicitly model the financial contagion mechanism, cannot adequately track the observed market return dynamics during the episode of financial crisis.

Figure 2.1. Smoothed States - Baseline

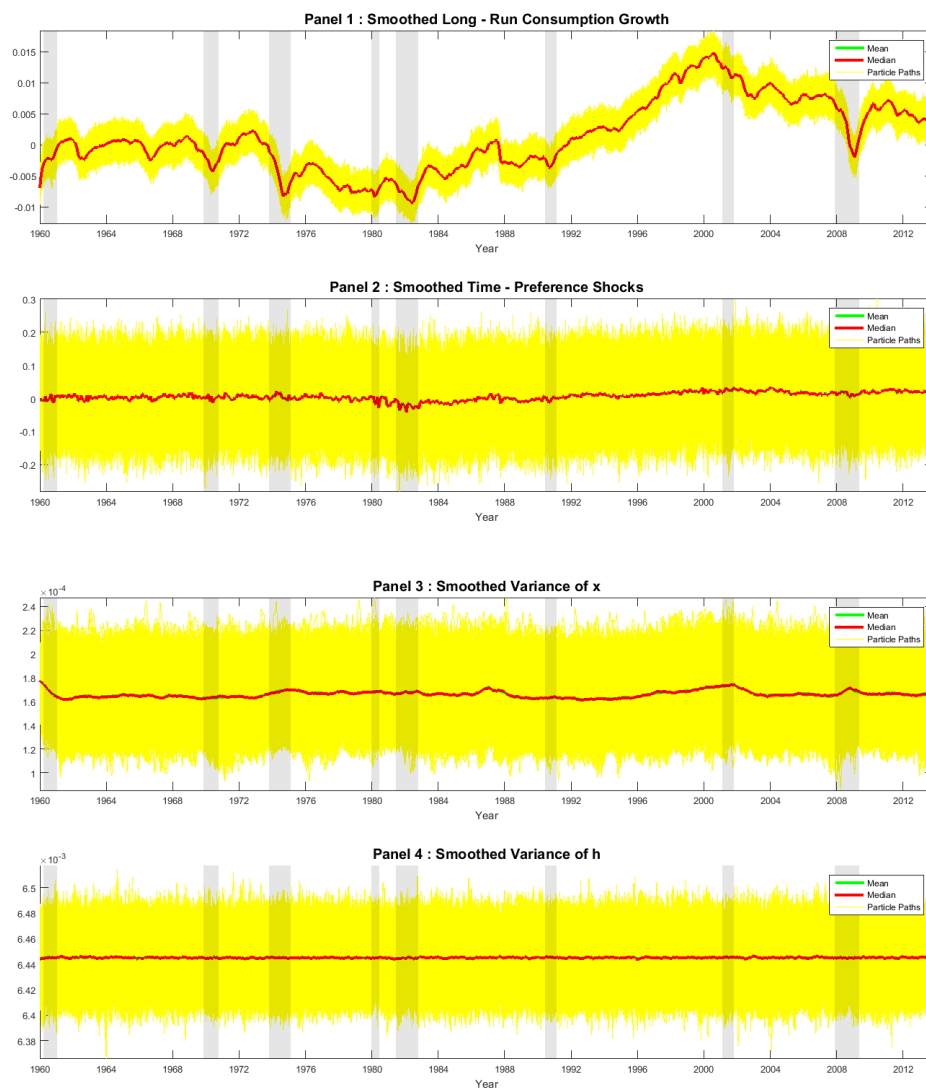
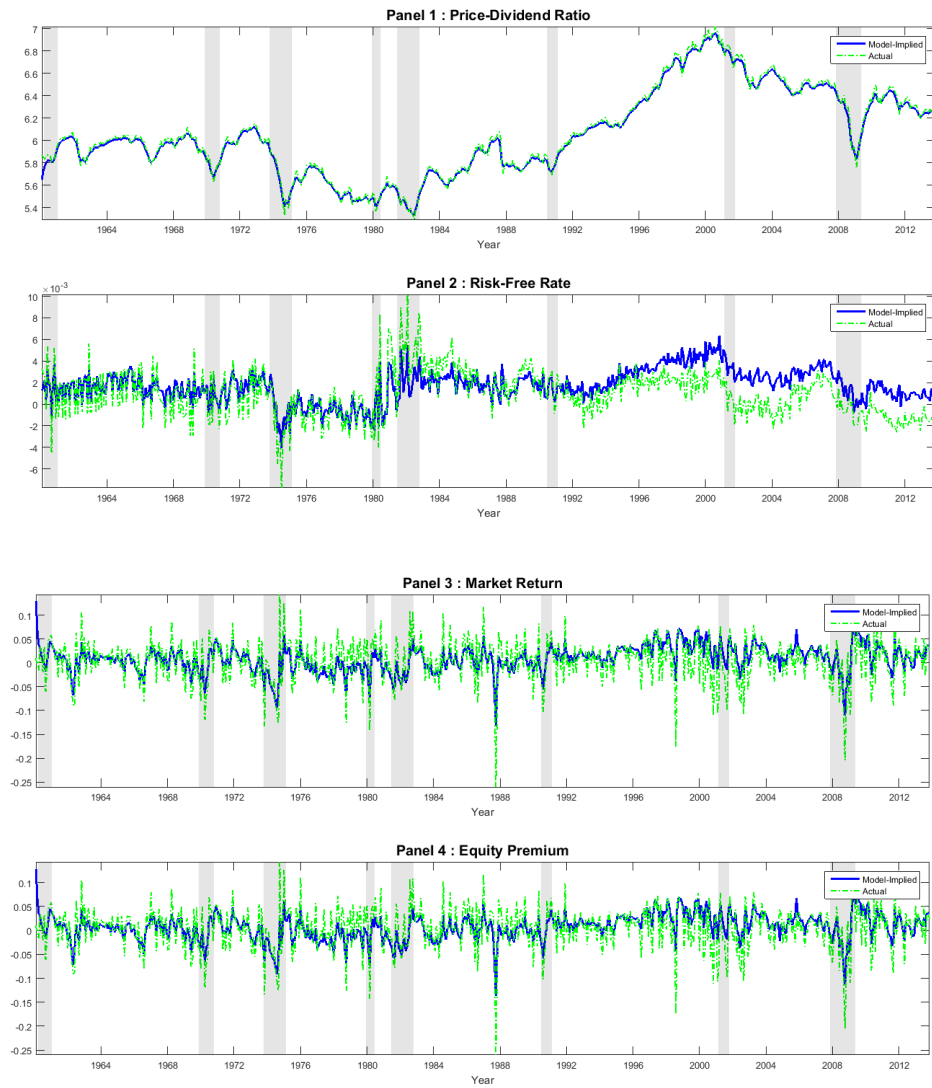


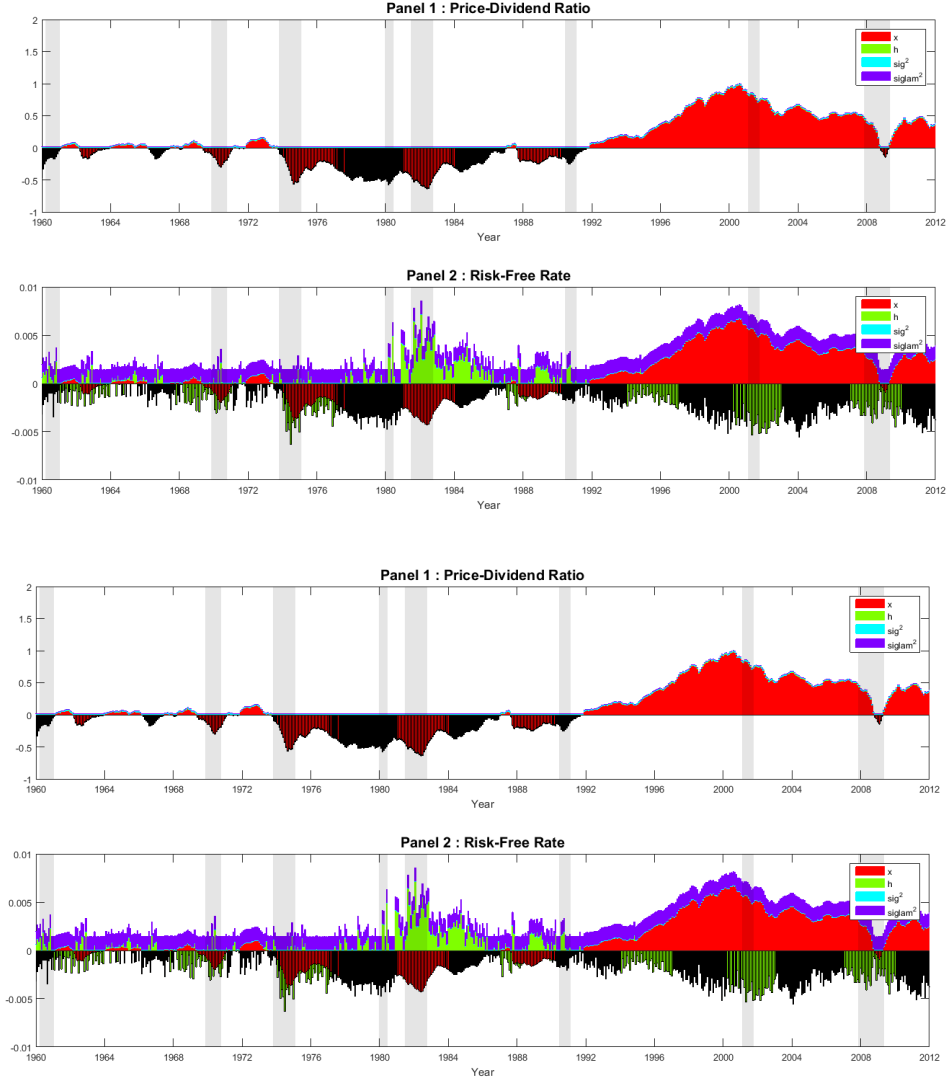
Figure 2.2. Model Fit - Baseline



Due to the lack of long-run risks studies that combine multiple channels in an integrated framework, the literature provides little guidance on the significance of the long-run consumption growth channel under the circumstance where non-consumption-related macroeconomic shocks are present. This study performs historical decomposition on the asset market variables and fills in the gap. As depicted in Panel 1 - 4 in Figure 2.3, the fluctuations in the price-dividend ratio, the market returns and the equity premium are overwhelming attributed to long-run consumption growth, whereas the contribution of time-preference shocks and the economic uncertainties are quantitatively negligible. In addition, a large fraction of the variation in the risk-free rate is explained by x_t , which indicates that long-run consumption growth is the most important component that drives the asset market fluctuations. However, these findings do not imply that the time-preference shock channel is redundant. First, Panel 2 suggests that, under the baseline model, time-preference shocks and valuation risks play a central role of accounting for the dynamic path of the risk-free rate. Second, recall that, in section 2.3, the “LRC Only” model seems unable to replicate the variation in the risk-free rate data. To confirm our conjecture, we estimate the latent state variables using the “LRC Only” specification and report the model-implied series in Figure 2.4. As shown in Panel 1, the “LRC Only” model is capable of tracking the observed price-dividend ratio even when the time-preference shocks channel is omitted. Nevertheless, the measurement errors associated with the market returns and the equity premium are quite sizable compared with those in the baseline model. More importantly, Panel 2 suggests that, without the time-preference shocks channel, the model-implied risk-rate not only exhibits very little variation throughout the sample, but also completely misses the target. In Table 2.5, we summarize the Mean Squared Error (hereafter, MSE) across all model specifications. We find that excluding the time-preference shocks channel would substantially weaken the model’s ability to rationalize the data. As shown in Table 2.5, for each individual variable (as well as for all variables combined), the “LRC Only” model returns larger MSE than does the baseline model. In particular, the MSE of the risk-free rate in the “LRC Only” model is twenty times as large as in the baseline case. Therefore, these pieces of empirical

evidence suggest that time-preference shocks are non-negligible factors that account for the asset pricing facts.

Figure 2.3. Historical Decomposition - Baseline



Beeler and Campbell (2012) point out that the long-run risks model tends to generate strong co-movements between consumption growth and the risk-free rate when IES is greater than 1, a pattern that is not observed in the U.S. data. In this study, we find that to augment the standard long-run risks model with the time-preference shocks channel would potentially help to reconcile the discrepancy. As depicted in Panel 2 of Figure 2.3, the observed move-

Figure 2.4. Model Fit - LRC Only

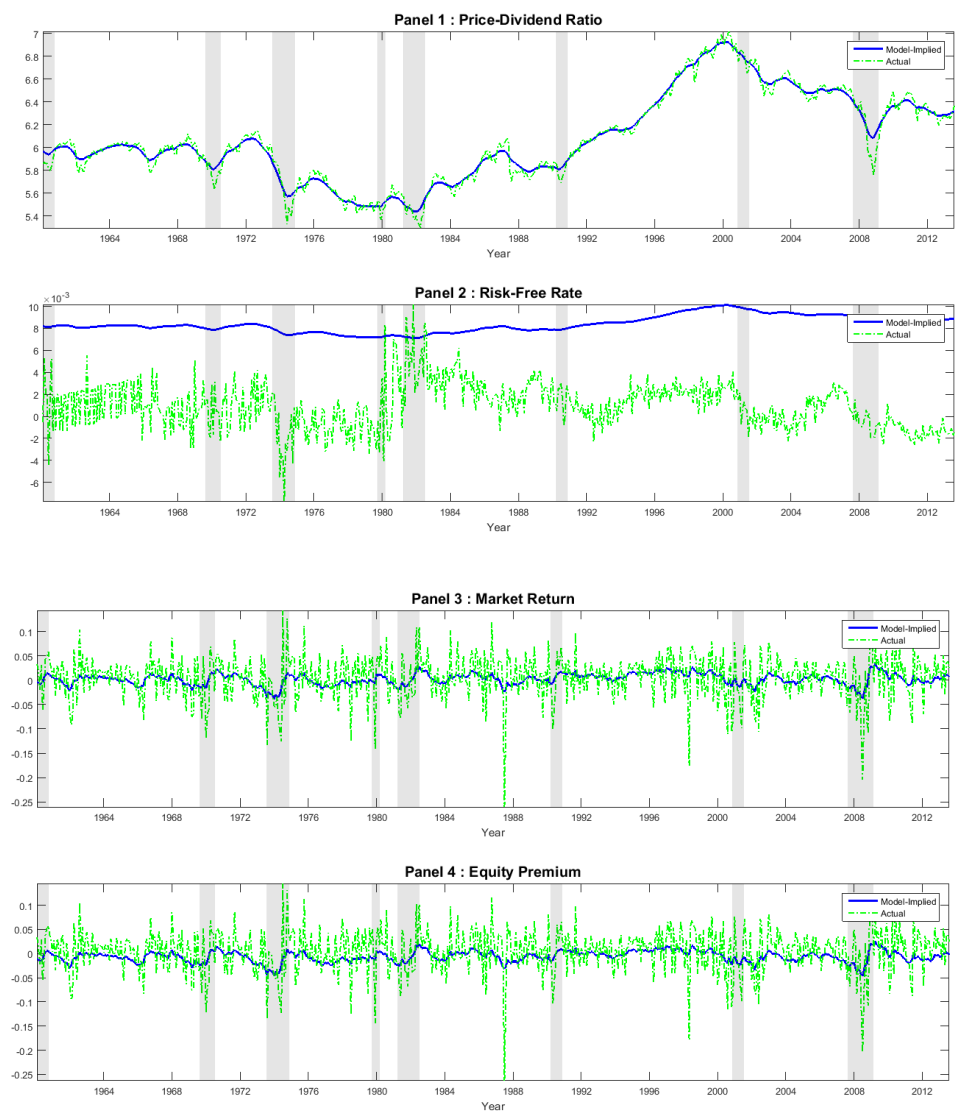


Table 2.5. Summary of Mean Squared Error

Model	z_d	r_f	r_m	All Variables
Baseline	0.0008	$2.70e^{-6}$	0.0009	0.0017
LRC Only	0.0032	$6.06e^{-5}$	0.0016	0.0049
TPS Only - Persistent	0.0612	$5.34e^{-4}$	0.0010	0.0627
Baseline - Persistent	0.0026	$8.87e^{-6}$	0.0039	0.0065

Note: First, “TPS Only - Persistent” refers to the “TPS Only” model with persistent time-preference shocks; and “Baseline - Persistent” refers to the baseline model with persistent time-preference shocks. Second, Table 2.5 does not include the measurement error of the realized equity premium, because the model-implied equity premium is computed as the model-implied market returns less the model-implied risk-free rate.

ments in the risk-free rate reflect a mixture of the effects of long-run consumption growth, time-preference shocks and the economic measure of uncertainties. The presence of the time-preference shocks weakens the tendency for consumption growth to move predictably with short-term real interest rate. For example, when long-run consumption growth rises rapidly during the 1990s, it induces a strong positive effect which would potentially push up the risk-free rate. However, such positive effect is largely offset by changes in agents' time-preferences, and the risk-free rate remains at a relatively low level. Given that time-preference shocks do not influence aggregate consumption growth, no strong co-movements between these variables would be generated under our model.

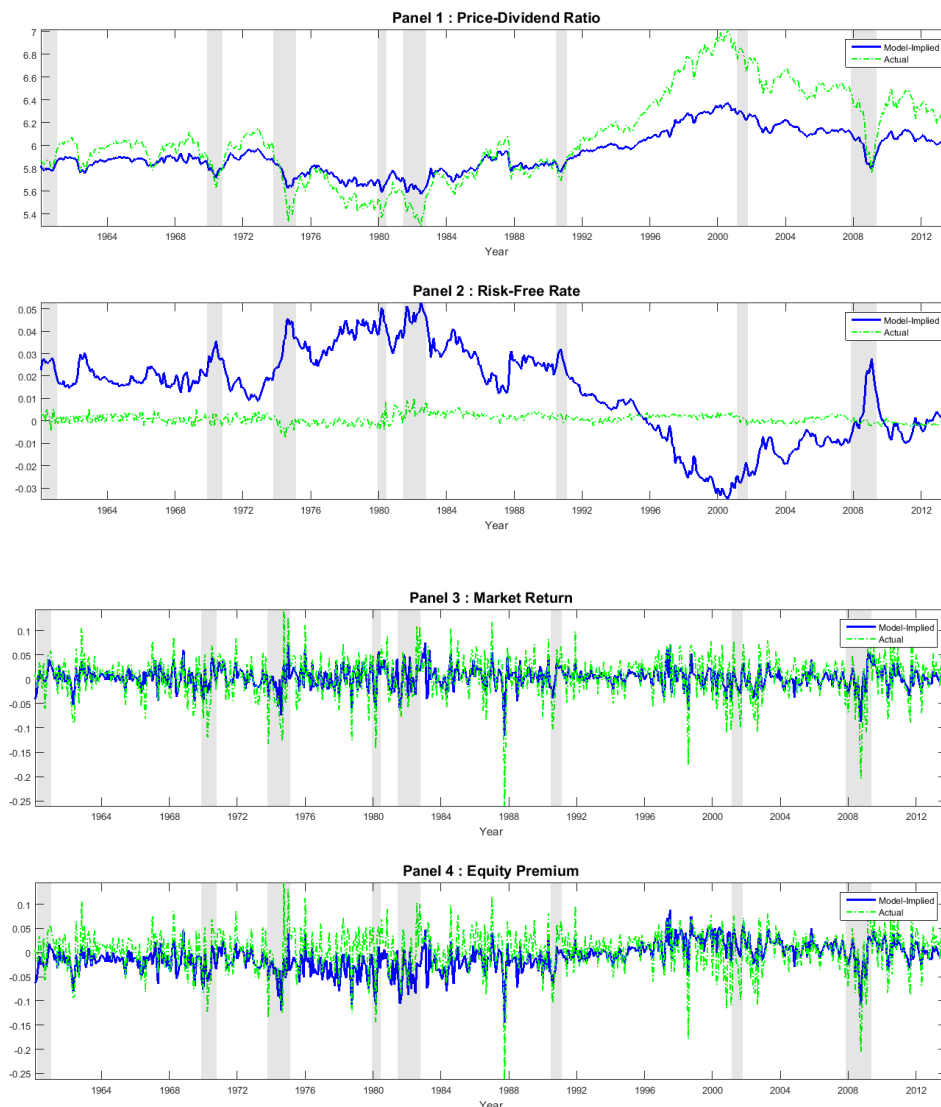
Another important issue this study attempts to address is whether changes in agents' time-preferences are persistent. Albuquerque, Eichenbaum and Rebelo (2013) report that the point estimate of the auto-regression coefficient on the time-preference shocks process is close to 1. Exploring two model specifications, however, we find (in section 2.3) that time-preference shocks seem to be near white noise. To assess the plausibility of our ρ_λ estimates, we conduct two experiments. In the first experiment, we set ρ_λ at 0.9 and repeat the two-step procedure to re-estimate the economic parameters and the latent states using the "TPS Only" specification.²⁵ We focus on persistent time-preference shocks in the "TPS Only" model for two reasons. First, while not exactly the same, this specification is quite close to the model proposed in Albuquerque, Eichenbaum and Rebelo (2013). Second, we seek to figure out whether the time-preference shocks channel by itself can explain the joint dynamics of the price-dividend ratio and the return variables.²⁶ As shown in Figure 2.5, in the absence of long-run consumption growth, the model not only fails to capture the persistent changes in the price-dividend ratio, but also implies the risk-free rate dynamics that is too persistent. Moreover, the MSEs associated with $z_{d,t}$ and $r_{f,t}$ are 0.0621 and

²⁵This experiment consists of two steps. First, we calibrate ρ_λ at 0.9 and estimate the rest of the deep parameters using GMM, the procedure of which is described in section 2.3. Second, taking the parameters as given, we extract the latent state variables using the procedure described in this section.

²⁶In principle, the Albuquerque, Eichenbaum and Rebelo (2013) model would rely on persistent time-preference shocks to account for the long swings of the price-dividend ratio dynamics.

$5.34e^{-4}$, respectively, which are seventy times and two hundred times greater than the test statistics in the baseline model.

Figure 2.5. Model Fit - TPS Only - Persistent Time-Preference Shocks



In the second experiment, we set ρ_λ at 0.9 but draw model inference from the baseline model. As shown in Figure 2.6, the contribution of time-preference shocks to asset market fluctuations increases as the process of time-preference shocks becomes persistent. According to Figure 2.7, however, the baseline model with persistent time-preference shocks do not correctly track the market returns. In the meantime, the MSEs of the price-dividend ratio

Figure 2.6. Historical Decomposition - Baseline - Persistent Time-Preference Shocks

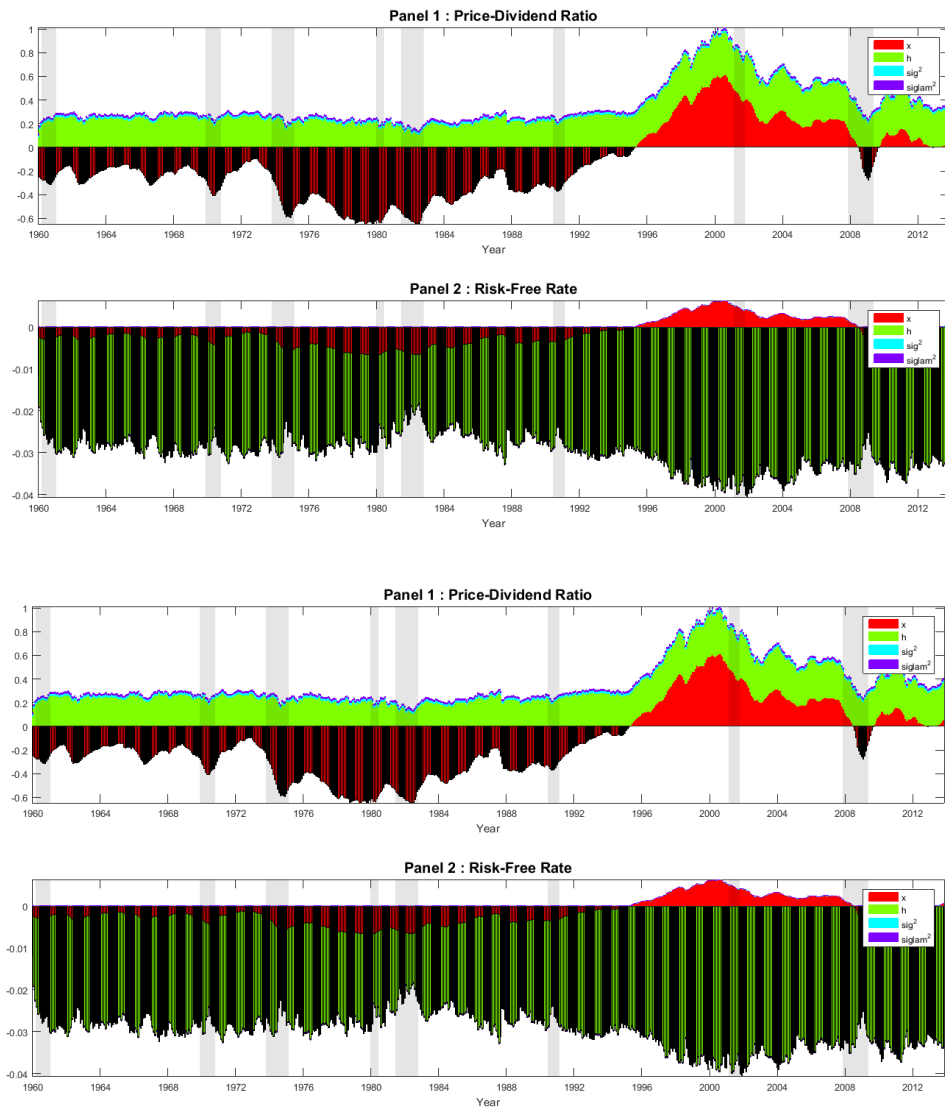
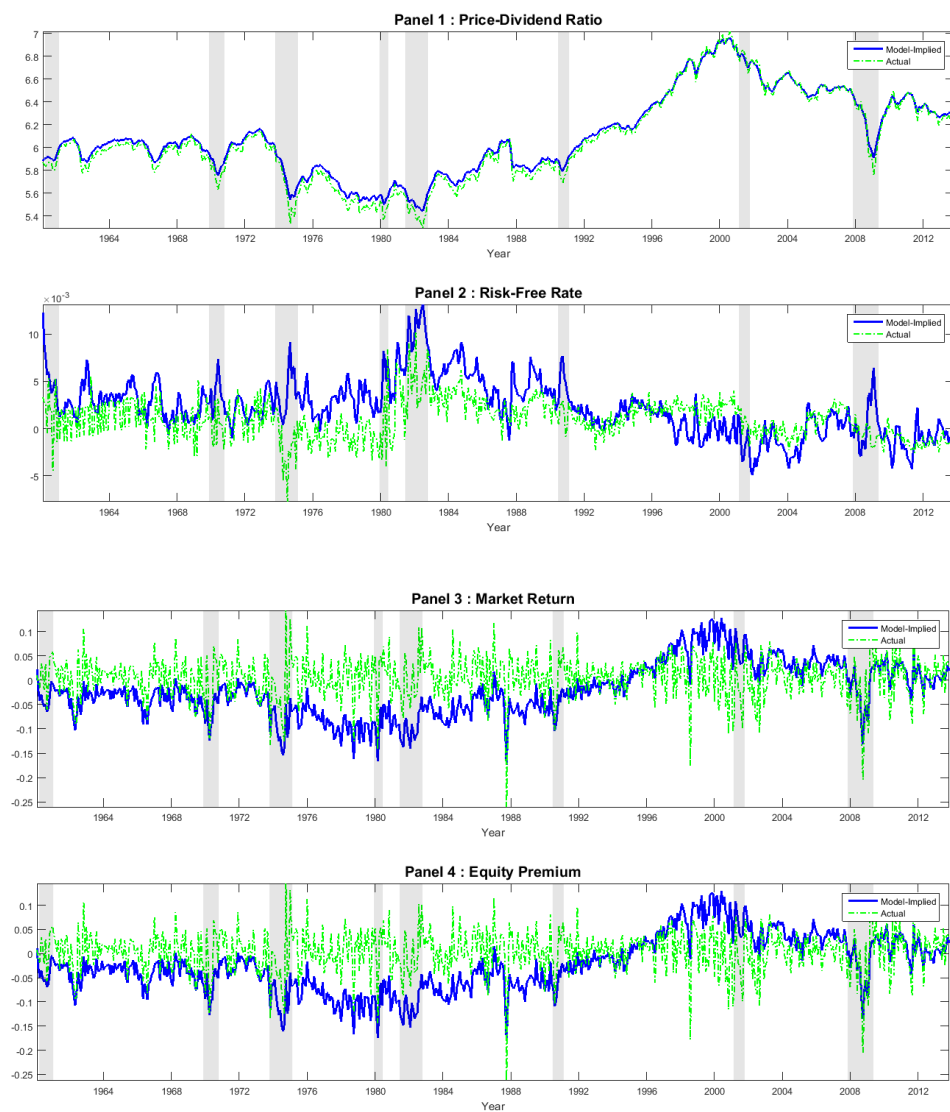


Figure 2.7. Model Fit - Baseline - Persistent Time-Preference Shocks



and the risk-free rate become four times larger than those in the standard baseline model. Therefore, forcing time-preference shocks to be persistent remarkably weakens the baseline model’s ability to replicate the data. And the combined evidence delivered by various experiments on model specifications suggests that neither the long-run consumption growth nor the time-preference shocks channel, by itself, is able to adequately account for the asset market phenomena; and the process of time-preference shocks seems unlikely to be persistent.

2.5. Conclusion

In this study, we exploit recursive preference specifications and propose a long-run risks model that is augmented with time-preference shocks. In the modeled economy, movements in asset market variables (such as price-dividend ratio, risk-free rate and equity returns) are driven by long-run consumption growth, time-preferences, and the relevant economic uncertainties.

Complementing the conventional moment-based methods, this study develops a two-step empirical approach to assess the model’s ability to rationalize the U.S. data. In the first step, we estimate the economic parameters using an efficient GMM estimator that is based on time-aggregated moment conditions. In the second step, we respect the non-linear nature of the exogenous processes and infer the latent state variables by approximating their probability densities using the particle filter and the particle smoother. In general, we find that our model is able to replicate the joint dynamics of the price-dividend ratio, the risk-free rate and the market returns without relying on an unreasonably large risk-aversion coefficient or IES parameter.

Empirical evidence, resting on a variety of experiments on model specifications and tests, highlights that both the long-run consumption growth and the time-preference shocks channels are important to understanding the asset pricing facts. On the one hand, it is found that, in the presence of non-consumption-related macroeconomic shocks, the importance of long-run consumption growth does not diminish. The fluctuations in the price-dividend ratio and market returns are still overwhelmingly due to the variation in long-run consumption

growth. On the other hand, time-preference shocks and valuation risks are found to be crucial determinants of the risk-free rate. In particular, the model that excludes the time-preference shocks channel cannot correctly track the risk-free rate.

While not delivering any direct policy implications, the findings in this paper can be potentially useful to the work that attempts to generalize the long-run risks model in the New-Keynesian Dynamic and Stochastic General Equilibrium (DSGE) framework. In recent asset pricing literature, an emerging stream of studies seeks to explore the welfare implications of monetary policy under DSGE models capable of explaining the asset market phenomena (for example, Diercks (2015)). As argued in this study, however, the standard long-run risks framework in a partial equilibrium model cannot properly account for the risk-free rate or the market returns. Therefore, future work might need to explicitly model the mechanism under which changes in agents' time-preferences are translated into a non-negligible source of asset market fluctuations.

Finally, it is worth pointing out that, while the estimated economic uncertainties contribute quantitatively little to asset price movements, the interpretation of the provided evidence is subject to caveat. In this study, the specification of the evolution of the volatility processes follows Bansal and Yaron (2004), and has been shown in the literature to be important to deriving the analytical solution to the equilibrium conditions. A noticeable drawback associated with this specification is that, the variance terms (σ_t^2 and $\sigma_{\lambda,t}^2$) might go negative when shocks of relatively large (but not necessarily extreme) values are drawn from the proposed distribution. So as to ensure the positivity of the estimated volatilities, in practice, researchers need either to impose restrictions on the parameters regulating these laws of motions or to winsorize the size of the innovations. Therefore, we can hardly rule out the possibility that the estimated low contribution of these risk measures is related to the additional restrictions imposed in the estimation procedure. Even though the stochastic volatility specification seems reasonable enough to mitigate the numerical issue, such setting in principle cannot deliver the analytical solution to the model. For example, Schorfheide, Song and Yaron (2014) revise the long-run risks model by employing the stochastic volatility

specification. But the solution to the equilibrium conditions relies on approximations of the stochastic volatility processes, which are in fact equivalent to those in the Bansal and Yaron (2004). To fully understand the asset market phenomena, it seems quite necessary to refine the existing modeling approach and further explore the effect of risks on asset price movements. This study, however, simply leaves it as an open question.

CHAPTER 3

FISCAL STABILIZATION AND THE CREDIBILITY OF THE U.S. BUDGET SEQUESTRATION SPENDING AUSTERITY

3.1. Introduction

Government debt escalated significantly following the Great Recession in five of the Group of Seven (G7) advanced economies. In France, Italy, Japan, the United Kingdom, and the United States, general government net debt, as reported by the International Monetary Fund, rose by about 30 to 50 percentage points of GDP between 2007 and 2015. That debt represented at least 80% of GDP in those G7 nations at the end of the period, an amount large enough to prompt concerns about the sustainability of governments' fiscal policies.

Of particular interest, the U.S. general government net debt nearly doubled, from about 40 percent of GDP in 2007, to 80 percent of GDP in 2012. This surge cannot be attributed solely to the cyclical increase of fiscal deficits in economic downturns, even with the especially deep 2008–09 Great Recession. It was also a byproduct of structural fiscal imbalances predating that contraction.

The Congressional Budget Office (CBO hereafter), a non-partisan federal agency, in a December 2007 report documented that the fiscal policy regime then in place implied an explosive path for the U.S. government debt.¹ A subsequent report by that same agency (CBO, 2010a), found that the Great Recession simply exacerbated the preexisting fiscal imbalances.

¹The CBO obtains the projections of fiscal variables implied by the prevailing policy regime with assumptions captured by an “alternative scenario”. The rather close correspondence of that scenario with current policy, rather than with “current law”, is documented more explicitly in analyses of the U.S. fiscal situation by the Peterson Foundation. See, for example, Peterson Foundation (2012).

Concerned with the negative long-run consequences of those structural imbalances, the Congress passed the Budget Control Act of 2011. An interesting feature of the law was the inclusion of a contingent clause that, starting in 2013, triggered a decade of government consumption expenditure reductions cumulatively totaling the equivalent to about 10% of nominal GDP in 2011. This provision has come to be known generically as “budget sequestration”, because its implementation entailed the revocation, or sequestration, of previously authorized expenditures.

The magnitude of the spending cuts has rekindled a debate in academic and policy forums about attempts to correct structural fiscal imbalances by reducing government expenditures and the effects of those cuts on economic activity. The result reported by Alesina and Perotti (1995) that those effects have been positive in several expenditure-based fiscal stabilization programs has been disputed, for example, in an International Monetary Fund (2010) study. Often forgotten in the heat of the discussion is the qualification, hinted at by McDermott and Wescott (1996), that the output effects of those programs depend critically on the extent to which economic agents expect the scheduled spending cuts to be enforced.

The goal of the present paper is precisely to provide an assessment of the credibility of the U.S. budget sequestration spending cuts with a novel methodology. It is one that in principle is applicable to other fiscal stabilization experiences, and for that reason potentially of interest in its own right.

The design of the methodology was guided by the implication of a wide class of economic models that show how different degrees of credibility of future spending cuts affect economic agents’ decisions and induce, as a result, a corresponding quantitatively distinctive response in key macroeconomic variables. It should be possible, therefore, to infer with well-accepted statistical tools which of the alternative credibility spending cuts scenarios are more likely to have accounted for the observed performance of those variables over the relevant period.

The methodology proceeds to make that inference by combining two approaches typically used in isolation in the economic literature: an “event study” approach, common in finance and exploited by Ramey and Shapiro (1998) to study the effects of government spending

policy shocks, and a “Business Cycle Accounting” (BCA) approach, originally developed by Chari, Kehoe, and McGrattan (2007) to study economic fluctuations within the analytical framework of general equilibrium models.

The motivation for incorporating an event-study perspective into the methodology was the prospect of obtaining a cleaner reading of the credibility of the spending cuts by limiting attention to evidence around the time of their initiation. The focus on a narrow window of time reduces the chances of contamination of responses of macroeconomic variables to that “policy event” from rare though sizable unanticipated shocks from other sources. This advantage was particularly handy, because the U.S. economy started to register the consequences of a large and persistent negative shock to oil prices in 2014. This development, as well as the chronology of events discussed in more detail later, buttress support for confining the evidence relevant for this paper to the years 2012 and 2013.

The BCA analytical framework was incorporated into the methodology adopted for this paper to inspire greater confidence in inferences obtained with a general equilibrium model. This is due to the BCA’s ability to accommodate various views of economic environment elements responsible for macroeconomic variables’ responses to shocks. On top of being endowed with these desirable properties, the BCA approach renders itself to a state-space representation of the economy that replicates the data exactly. This feature, along with the event-study approach, was key to making inferences about the credibility of alternative budget sequestration spending cuts scenarios with well-accepted likelihood-based techniques.

Implementation of this methodology surmounted three principal empirical challenges. First, measurement issues, which were addressed by treating the data with the “private sector economy” approach suggested by Gomme and Rupert (2007) and by introducing in the model economy an external-like sector in the manner proposed by Trabandt and Uhlig (2011). Secondly, the need to take into account the transitional dynamic effects of a permanent increase in the capital income tax rate scheduled to become effective in 2013 as a result of legislation enacted in 2010. Finally, a lack of consensus about the magnitude of two macroelasticities controlling the size of those transitional effects—the intertemporal

elasticity of substitution in consumption and the labor supply Frisch elasticity, resolved by assessing the credibility of alternative spending cuts scenarios for several combinations of values of those parameters.

The main finding of the paper is that, for all those combinations and by the standards of the maximum likelihood criterion, the budget sequestration spending cuts scheduled for 2014 and beyond enjoyed little, if any credibility during the relevant 2012–2013 event-study window.

For the reasons hinted at earlier, that is a finding that should be kept in mind before drawing policy conclusions from discrepancies between the predicted and observed outcomes of attempts to correct fiscal imbalances with spending austerity. The potential contribution of this paper along this dimension can be best illustrated by a brief discussion of the output effects, precisely, of the budget sequestration spending cuts. According to Cashin, Lenney, Lutz, and Peterman (2017), they negatively affected the level of economic activity. The finding in this paper suggests that the magnitude of those effects was determined at least in part by the lack of credibility of the spending austerity. Put differently, the responses of output and other macroeconomic variables could have been qualitatively or quantitatively different were the prospect of spending cuts credible.

More generally, the results reported by studies evaluating the responses of macroeconomic variables to expenditure-based fiscal consolidation programs might be misinterpreted absent a formal assessment of the credibility of the prescribed spending cuts. This paper proposes a methodology suitable for such an assessment. Examination of other fiscal austerity programs with a properly adapted version of the methodology might shed further light on the policy debate generated by program outcomes that do not always coincide with those predicted by theoretical or empirical considerations.

The rest of the paper is organized as follows. Section 3.2 reviews background material, chronology of events, and measurement issues that motivated many of the assumptions and details of specification of the model presented in section 3.3. Section 3.4 discusses first intuitively, and then in more detail, the adaptation of the BCA approach and the statistical tools

that the paper exploits to make inferences about the credibility of the budget sequestration spending cuts. Section 3.5 reports the findings. Section 3.6 concludes. An appendix scrutinizes further with Bayesian techniques the findings of the paper for a particularly relevant pair of values of the two macroelasticities identified above.

3.2. Background Material

3.2.1. The Budget Sequestration: Relevant Details and Timeline of Events

Background information on the institutional features of the budget sequestration and a timeline of events leading up to it provides context for several of the modeling choices made in the paper to gauge the credibility of spending cuts prescribed by U.S. legislation enacted in 2011.

Table 3.1 provides a background for the account below of the historical circumstances, not exempt of dramatic twists, that ultimately ended in the cuts. It identifies by date key developments, along with brief comments regarding their significance relative to the motivation and purpose of this paper.²

The road to the budget sequestration started when the Treasury requested on January 6, 2011, that the Congress authorize an increase in the debt ceiling, necessary to roll over the outstanding debt as well as to finance current fiscal deficits. The U.S. government can borrow to finance a revenue shortfall relative to expenditures so long as it doesn't exceed the "debt ceiling" explicitly authorized by Congress. The authorization step is usually a formality, as it simply provides the U.S. Treasury the means to pay for government spending Congress previously approved. At the beginning of 2011, however, a large number of lawmakers were reluctant to rubber stamp the authorization as they had routinely done in the past. They indicated concern with the rapidly growing government debt the CBO had projected.

These legislators demanded, therefore, that any increase in the debt ceiling be accompanied by fiscal deficit reduction measures to blunt debt growth. There was, however,

²More detailed chronology can be found in <http://www.cnn.com/2011/POLITICS/07/25/debt.talks.timeline/>

Table 3.1. Timeline of Events/Developments Leading up to the Budget Sequestration Cuts

Date	Event / Development
January 6, 2011	U.S. Treasury requests Congress raise debt ceiling. Routinely granted authorization withheld by lawmakers concerned by explosive debt path projected in a June 2010 CBO report.
July 14, 2011	Debt ceiling nearly reached. Potential inability of U.S. Treasury to meet its obligations prompts Standard & Poor's credit rating agency to place U.S. government debt on "CreditWatch with negative implications."
August 2, 2011	Last-minute deal allows Congress to pass the Budget Control Act, reducing fiscal deficits in two staggered installments. Second installment would trigger a budget sequestration procedure and sizable automatic spending cuts starting in 2013 if a bipartisan Joint Committee can't agree on fiscal reduction measures by January 15, 2012.
November 21, 2011	Joint Committee admits deal to avert automatic spending cuts not possible.
Year 2012	President and Congress vow to find compromise to prevent activation of automatic spending cuts. With some temporary tax cuts expiring, deliberations create opportunity for another last-minute agreement.
January 1, 2013	American Taxpayer Relief Act passed, postponing automatic spending cuts by just two months.

considerable disagreement about the form of those measures, and grueling negotiations to resolve the differences put the U.S. at the brink of a sovereign debt default. A last-minute agreement avoided that outcome. The Budget Control Act was signed into law on August 2, 2011. The law created a bipartisan Joint Select Committee on Deficit Reduction of lawmakers, assigned the task of finding fiscal deficit reduction measures totaling \$1.5 trillion (equivalent to about 10% of nominal GDP at the time) over the following decade.

The Budget Control Act included a provision stating that if the Joint Committee failed to propose reductions and Congress subsequently failed to act on deficit cuts totaling at least \$1.2 trillion by January 15, 2012, spending caps on discretionary budget authority would be imposed in the cumulative amount just mentioned, starting in January 2, 2013, and lasting through fiscal year 2021.

In practice, this contingent clause would accomplish its \$1.2 trillion fiscal stabilization goal (inclusive of savings in interest payments on government debt) either with the deliberate measures suggested by the Joint Select Committee or, in their absence, with automatic spending cuts evenly split between across-the-board between discretionary defense and non-defense programs.

The lower spending caps stipulated wouldn't legally apply to previously authorized but not yet materialized spending, an institutional difficulty identified by the CBO (2013, p. 31). The Budget Control Act got around that technicality by ordering the application of "budget sequestration" procedures that revoked (or sequestered) de facto preexisting authority to spend, in the amount needed to conform to the lower caps. This is why the paper refers to all the spending cuts implied by the contingent clause of the Budget Control Act as budget sequestration cuts, even if strictly speaking, sequestration applied only to the budget items that the CBO had noted.

In order to trust that the model below is an adequate abstract representation of the actual economy, it is important to note that the sequestration cuts would affect public sector payrolls only through furloughs of limited duration and scope. Given the lack of a measurable effect on public sector employment, this feature of the legislation turned out to

be convenient for circumventing the measurement difficulties hinted at in the introduction. It allows consideration, without loss of realism, of a model economy in which the government doesn't make any contribution to value added and whose spending is captured by the quantity of goods and services that it removes from the private sector.

More relevant is the observation that, if implemented in full, budget sequestration would reduce discretionary spending to the lowest level on record as a share of GDP, according to CBO (2012b) estimates.³ It didn't seem plausible to treat spending cuts that reduced government consumption and investment that much as a manifestation of one of the many fluctuations that the macroeconomic variable typically exhibited under the existing policy regime. A more appropriate interpretation would appear to be that budget sequestration, if triggered, would represent a decade-long policy regime shift.

For that reason, the model treats the ratio of government absorption of goods and services to private sector output as consisting of two components, rigorously presented in section 3.3.3.1. The first, an exogenous stochastic component, is meant to capture run-of-the-mill historical fluctuations of that ratio around a long-run mean. The second, a deterministic component, is meant to capture the temporary policy regime change that sequestration would eventually bring.

Back to the chronology of events, the strong incentive to reach an agreement on a fiscal deficits reduction package introduced in the Budget Control Act by the rather blunt budget sequestration threat didn't seem to be working as intended, however, when on November 21, 2011, the Joint Committee announced that, *"after months of hard work and intense deliberations"*, it had come to the conclusion that it wouldn't be possible to reach an agreement on an alternative fiscal deficit reduction package before the January 15, 2012 deadline.

That development was sufficiently significant to perhaps induce the private sector to expect the budget sequestration to be effectively launched a year later and for the private sector to adjust its behavior accordingly in 2012. There are also good reasons, however,

³More specifically, in Table 3.1 of the cited CBO report, discretionary spending at the end of the sequestration period, in 2021, is projected to represent 5.7% of GDP, the lowest level observed since at least 1972.

to be skeptical that that was the case. First, that the Congress would eventually act when faced with sizable cuts eventually impairing the ability of government agencies to adequately perform core functions. Second, negotiations regarding extension of temporary tax cuts enacted in 2001 and 2003 and due to expire in 2012 were viewed as offering legislators a golden opportunity to come up with alternative deficit reduction measures that met the necessary conditions to cancel, or at least suspend budget sequestration. Such speculation may have been reinforced by repeated public statements from Congress and even the President on their determination to find a compromise.⁴

There is, therefore, the distinct possibility that, as of the end of 2012, households and businesses were still dismissing materialization of the policy regime change represented by the budget sequestration. But that may have changed rather dramatically when Congress passed the American Taxpayer Relief Act in early 2013 and modified the tax code as expected, but failed to take any substantial action with respect to sequestration, other than postponing its implementation by two months, from January 2 to March 1, 2013. The law's passage may have convinced households and businesses that the budget sequestration was no longer a distant, unlikely event.

Given the timing of events and circumstances surrounding them, the assessment of the credibility of the budget sequestration cuts required establishing when it was most likely that economic agents would incorporate those cuts in their decisions, as early as 2012 or when they were effectively triggered a year later, in 2013. The observation that an event study approach is particularly well-equipped to confront this challenge was one of the reasons, therefore, to have incorporated it to the methodology developed in this paper.

Finally, another detail with implications for the evidence that will be examined in the paper is that, as mentioned in the introduction, the Health Care and Education Reconciliation Act of 2010 introduced an additional tax of 3.8% on net investment income—a form of capital income taxation—that would enter into effect in 2013, precisely at the same time

⁴In fact, according to press reports, the Department of Defense, one of the federal agencies that would be hit particularly hard by the spending cuts, wasn't making any contingent plans to deal with them as late as September 2012.

that the circumstances described above triggered the budget sequestration spending cuts under study.

3.2.2. Spending Cuts in Real Terms

The goal of fiscal stabilization programs is typically to prevent the government debt from ballooning out of control relative to the size of the economy. Their specific measures must be designed, therefore, with the target of reducing fiscal deficits by a certain amount in *real terms*. Given that the Budget Control Act represented an attempt to correct U.S. fiscal imbalances, such a target must have dictated the size of the spending cuts it prescribed. What should be assessed, therefore, is the credibility of the size of those cuts in real terms targeted by that legislation.

Unfortunately, information about that target is missing from the Budget Control Act or any of the other many official records examined for the purpose of this project. Moreover, as indicated in the previous section, that law lowered existing caps to nominal government spending, with the effect of implying the spending cuts of interest for this paper only in nominal terms.

For the purposes of this project it was necessary, therefore, to convert those nominal spending cuts into ones in real terms. To be informative, any procedure used to that end must start out from a reliable sequence of the nominal spending cuts implied by the Budget Control Act. Fortunately, such a sequence can be readily constructed from the data provided in an analysis of that legislation by the CBO (2013), as summarized in the second column of Table 3.2.

The third column of the table shows the result of dividing the figures in the second one by the private sector nominal GDP in 2012, the last one that economic agents could observe before the initiation of the budget sequestration in 2013.⁵ This algebraic operation simply rescales the sequence of nominal spending cuts to that that would be observed in an economy

⁵Nominal private sector GDP in 2012 was \$14,126 billion, as estimated with the “private sector output” methodology mentioned in the paper, using the relevant data from the U.S. National Income and Product Accounts (NIPA) prepared with the comprehensive methodological revision introduced in 2013.

Table 3.2. Annual Budget Sequestration Spending Cuts

Year	\$ Billion (*)	\$ Billion Normalized to 2012 Nominal Output	Targeted Spending Cuts	
			Per Unit of Output in Real Terms	Per Unit of Output in Real Terms, Detrended
2013	35	0.0025	0.0024	0.00238
2014	75	0.0053	0.0051	0.00491
2015	85	0.0060	0.0057	0.00534
2016	89	0.0063	0.0058	0.00538
2017	90	0.0064	0.0058	0.00523
2018	90	0.0064	0.0057	0.00502
2019	89	0.0063	0.0055	0.00478
2020	88	0.0062	0.0053	0.00454
2021	87	0.0062	0.0051	0.00431

(*): Congressional Budget Office (2013), p. 10 and Table 1-5, p. 27.

with a nominal GDP level of one in 2012, but preserving their size relative to the actual level of that variable in the data. The motivation for this intermediate step is that that level of output at its steady-state in the model economy is normalized, precisely, to the value of one.

The normalized spending cuts in the third column of Table 3.2 are still in nominal terms, because they were obtained making abstraction of inflation. In order to estimate their magnitude in real terms, it was necessary to make assumptions about the evolution of the inflation rate over the period the budget sequestration would be in effect. It seemed sensible to conjecture that the legislators that enacted the Budget Control Act implicitly prescribed the nominal spending cuts they did because they projected that their size in real terms would be enough to correct fiscal imbalances in an environment free of inflationary surprises.

In other words, it is legitimate to assume that U.S. lawmakers were counting on the ability of the Federal Reserve to keep the inflation rate rather close to its 2% annual target for the budget sequestration spending cuts to deliver their underlying fiscal stabilization goal. It seems fair to conclude, therefore, that the Budget Control Act was passed in the understanding that inflation would erode the value of the spending cuts it prescribed at the annual rate of 2%.⁶ From a mechanical point of view, this assumption is equivalent to dividing those cuts by a price index that grows exponentially at the annual factor of 1.02 from its 2012 base year value of 1. The application of this “gross down” factor to the normalized spending cuts in Table 3.2 resulted in the sequence of spending cuts per unit of output adjusted by projected inflation, that is, in real terms, recorded in the fourth column of that table.

Finally, for the reasons discussed in section 3.3, all real variables of the actual economy that can be represented as a proportion of GDP were detrended by the secular 2% annual growth rate of real output implied by the calibration of the model. Since government expenditures meet that condition, methodological consistency required to detrend the budget sequestration spending cuts in the same manner. The results of that detrending procedure

⁶This assumption is consistent with the projections of several inflation rate indicators that can be found in the same CBO report cited as the source of the nominal spending cuts implicitly mandated by the budget sequestration.

is captured in the last column of Table 3.2. The figures in that column correspond to the spending cuts in real terms per unit of output implicitly *targeted* by the Budget Control Act, as “reversed-engineered” from their nominal counterparts with the assumptions described above.

To avoid misunderstandings, it is important to emphasize that the paper doesn’t assume that economic agents expected those targeted spending cuts in real terms to be actually enforced. In fact, the purpose of the paper is precisely to establish the credibility of those cuts from the perspective of households and businesses, as captured by the effects of their unobserved decisions on observed macroeconomic variables during the relevant event-study window.

Furthermore, it should be kept in mind that, although presented for convenience of exposition in terms of per unit of output, in the empirical implementation of the model the targeted spending cuts in real terms identified in the last column of Table 3.2 will be treated as the *absolute value* of those cuts, because they will correspond to the absolute deviation of government consumption from its steady-state level in a model economy whose steady-state level of output will be calibrated to the value of 1.

The Budget Control Act didn’t stipulate spending caps past the year 2021, so it didn’t impose any legal restrictions on the level of government absorption of goods and services as a share of GDP in the long run. The value of this ratio in the long run is needed, however, because the steady state equilibrium of the model economy will be an important reference for the empirical implementation of the BCA approach adopted by the paper. The developments summarized above suggest that the budget sequestration was a fiscal stabilization measure of last resort and, as such, didn’t have any lasting effects in the long-run government absorption of goods and services to private sector output ratio, as measured in section 3.3.4 of the paper.

3.2.3. Measurement Issues

Given the limited data inherently available to a methodology that, as the one adopted for this paper, peeks at the evidence with an event-study perspective, it seemed important

to minimize the imprecisions in the assessment of the credibility of the budget sequestration spending cuts introduced by the measurement errors pointed out by Gomme and Rupert in their already cited paper.

It would take a long detour to go over the measurement inaccuracies, potentially severe, that those authors trace to procedures that take the national accounts at face value in the attempt of mapping macroeconomic variables represented in the standard neoclassical growth model into their empirical counterparts. For the purpose of this paper, it suffices to say that some of those inaccuracies can be mitigated with a version of the neoclassical growth model in which all value added is generated by private sector firms. The empirical counterpart of this output concept in the model economy is obtained by subtracting from GDP in NIPA the value added by the general government in the process of producing non-market goods and services.

It is important to emphasize that the Gomme-Rupert “private sector economy” approach will not be an obstacle to make inferences about the credibility of the budget sequestration cuts because, as mentioned earlier, they fell mostly on the government absorption of goods and services produced by the private sector, rather than on the value added by the government, a large fraction of which is just the compensation of the labor services provided by government employees.

The data necessary to obtain the historical series of private sector output, in a manner consistent with the treatment of government economic activities in NIPA, are available at an annual frequency only since 1977. The analysis in this paper uses therefore data from that year until 2013, the last year providing relevant evidence from the event-study perspective adopted by the paper. A thorough discussion of the steps required to make the data for the 1977-2013 period consistent with the conceptual entities in the model are rather involved and would detract from the main focus of the paper. Readers interested in the details will be able to find them, however, in Kydland and Zarazaga (2016), who applied an entirely analogous procedure in the process of answering a different fiscal policy question.

3.3. The Model Economy

Taking into account that the paper incorporates a BCA approach to the methodology proposed for inferring the credibility of the budget sequestration spending cuts, it seemed sensible to respect the principle generally followed by previous implementation of that approach that the long-run features of aggregate models should be consistent with the balanced growth facts documented by Kaldor (1961). Accordingly, preferences, technology, and government policies have been restricted to the types that are consistent with balanced growth, as characterized by King, Plosser, and Rebelo (1988a, b).

All real variables were obtained by dividing their nominal counterparts by the price index of non-durable goods and services. This procedure guarantees that all investment-specific technological progress can be transformed, with the appropriate choice of production function, into labor-augmenting technological progress, the only kind of technological progress consistent with balanced growth, as discussed in King, Plosser, and Rebelo (1988a, b).

Also, when applicable, all real variables are represented in terms of per population 16 years of age and over and detrended by the long-run growth rate of total factor productivity. This procedure typically removes the secular trend from the variables of interest. The exception is the fraction of available time that households are at work in the private sector. The rising trend exhibited by this labor input series, driven by an increasing participation of women in the labor force and demographics, was removed with the procedure proposed by Kydland and Zarazaga.

In other words, the variables of the actual economy were transformed to those corresponding to an economy without growth with the appropriate detrending procedures. As is well known, this transformation is without loss of generality, because it displays the same transitional dynamics as the original economy with secular deterministic growth, but is more convenient to work with when, as in the case of this paper, the technique for computing the equilibrium allocations involves Taylor expansions of the first-order conditions around the deterministic steady-state.

3.3.1. The Typical Household's Choice Problem

The model economy is assumed to be inhabited by an infinitely-lived household, which stands for the large number of them present in the actual economy and whose preferences can be ordered by a time-separable Constant Frisch Elasticity (CFE hereafter) utility function defined over infinite streams of consumption $\{c_t\}_t^\infty$ and the fraction of available time devoted to work $\{h_t\}_t^\infty$. In addition to being consistent with balanced growth, this utility function is the only one that allows consumption and leisure to be non-separable within periods without at the same time tying the value of the *marginal-utility-held-constant* labor supply real wage elasticity—the so-called Frisch elasticity—to that of the *labor-held-constant* intertemporal elasticity of substitution in consumption (IES hereafter) and to the fraction of time devoted to work. Given the purpose of this paper, the flexibility of this utility function for specifying different values for the Frisch elasticity and the IES was important for conceptual and computational reasons.

The conceptual reason is that the strength of the response of endogenous macroeconomic variables to a fiscal policy change, such as the one studied in this paper, is controlled not only by the credibility inspired by the policy, but also by the value of the two macroelasticities just mentioned. Given the considerable disagreement about those values prevailing in the profession, it seemed prudent to explore the credibility of the budget sequestration with a utility function consistent with combinations of them that would be disallowed by the one-to-one correspondence between the value of the IES and that of the Frisch elasticity implied by the alternative popular Constant Elasticity of Substitution (CES) specification for the utility function, also consistent with balanced growth.

The computational reason for the adoption of a CFE utility function specification is that the unavoidable approximation errors introduced by the perturbation method used to compute the private sector's decision rules are likely to be compounded by utility function that implies, as the alternative CES specification does, that the Frisch elasticity varies with the fraction of available time devoted to market activities and is different, therefore, at the steady state and out of it.

Accordingly, the stand-in household is assumed to solve the following maximization problem:

$$\underset{\{c_t, h_t, k_{t+1}\}}{Max} E \sum_{t=s}^{\infty} [\beta(1+\gamma)^{1-\sigma}(1+\eta)]^t \frac{c_t^{1-\sigma} [1 - \kappa(1-\sigma) h_t^{1+\frac{1}{\varphi}}]^\sigma - 1}{1-\sigma} \quad (3.1)$$

subject to the following constraints:

$$c_t + (1 + \tau_t^x) x_t = (1 - \tau_t^h) w_t h_t + r_t k_t - \tau^k (r_t - \delta) k_t + n i_t + \tau_t \quad (3.2)$$

$$x_t = (1 + \eta)(1 + \gamma) k_{t+1} - (1 - \delta) k_t \quad (3.3)$$

$$1 = l_t + h_t \quad (3.4)$$

$$h_t = h_t^{pr} + h_t^{pu} \quad (3.5)$$

$$\text{government policies} \quad (3.6)$$

The objective function in (3.1) is the expected discounted value of a utility function in the CFE class, where $\beta > 0$ is the discount factor; η is the working age population annual growth rate; γ is the annual growth rate of total factor productivity; t is a time index; c_t is detrended consumption per working age person; h_t is the fraction of available time the representative household allocates to work in the market; $\sigma > 0$ is the inverse of the IES; $\kappa > 0$ is a parameter that controls the household's valuation of consumption relative to leisure; and φ is the constant Frisch elasticity of aggregate labor supply.⁷

Equation (3.2) is the household's budget constraint, where x_t is gross private domestic investment; w_t is the wage rate in terms of consumption per unit of the available time the stand-in household devotes to work; r_t is the rental price of period t private sector capital; k_t , τ^k is the tax rate on income from that capital; δ is the depreciation rate; and τ_t denotes lump-sum transfers (taxes if negative.) The three symbols not discussed yet, τ_t^x , τ_t^h and $n i_t$, introduce in the model three of the four “wedges” that will implement the BCA approach

⁷Recall that the multiplication of the discount factor β by the factor $(1 + \eta)(1 + \gamma)^{(1-\sigma)}$ is the result of removing from aggregate consumption the deterministic annual secular growth rate $(1 + \eta)(1 + \gamma)$.

incorporated to the methodology for making inferences about the credibility of the budget sequestration. In particular, τ_t^x and τ_t^h play the same role as in CKM, by determining what those authors refer to, respectively, as the *labor wedge*, $1 - \tau_t^h$, and the investment wedge, $1/(1 + \tau_t^x)$.

As in CKM, the wedges summarize in convenient “auxiliary” variables the presence of not explicitly modeled frictions that distort equilibrium allocations relative to those of a frictionless model economy. For example, the investment wedge τ_t^x might be interpreted as capturing output losses or gains associated with the relaxation or tightening of both, liquidity constraints on consumers and/or financing restrictions on firms. It can be verified that also this wedge will capture, through its effects on intertemporal equilibrium conditions, changes in the effective real interest rate—the effective real return on capital in the model—induced by variables not explicitly included in the analysis.

The variable ni_t stands for net imports and captures the net exports component of aggregate demand that CKM lumped together with a government consumption wedge. It could be interpreted therefore as a stochastic *external sector wedge*, introduced in the minimalist manner proposed by Trabandt and Uhlig. These authors introduced this wedge to mitigate the lack of correspondence between the otherwise closed economy neoclassical growth model and the U.S. economy, whose economic interactions with the rest of the world are considerably more challenging to model and parameterize explicitly.

The empirical implementation of the model will take into account that in balanced growth the ratio of ni_t to output should be characterized by a stationary stochastic process with unconditional mean niy . Section 3.4.2 will provide further details about this process, as well as of those governing the evolution over time of the labor wedge τ_t^h and of the investment wedge τ_t^x .

Equation (3.3) states the evolution over time of the detrended capital stock that the household rents to private firms which, for consistency with the NIPA methodology, excludes the public sector capital stock. This law of motion links the private sector capital stock available for production at the beginning of a period, k_t , with the households’ investment

decisions during that same period, x_t , and with the private sector capital stock that will be available at the beginning of the following period, k_{t+1} .⁸

Equation (3.4) states the time constraint that the stand-in household can distribute its total available time, normalized to 1, among non-market activities, l_t , (generically labeled as “leisure”) and work in the marketplace, h_t .

Equation (3.5) states that the household can allocate the time it devotes to work between private sector firms, h_t^{pr} , and public sector agencies (inclusive of government-owned enterprises), h_t^{pu} . Note that for consistency with the standard treatment of labor input in the neoclassical growth model, the empirical counterpart of variable h_t is the fraction of time actually worked, not just paid. The data were therefore adjusted to exclude the time for which workers were paid but not actually working, because they were on vacation, sick leave, etc.

Notice also that without the uncommon explicit distinction between the time households allocation to work in the public and private sectors, the computation of the model output would have been unfeasible with the private sector output methodology approach adopted by this paper.

3.3.2. Private Sector Firms’ Maximization Problem

There are two kinds of firms that produce output in the stationary economy without growth and without a government final good: private firms and government enterprises. As pointed out by Gomme and Rupert in the paper repeatedly mentioned, the decisions of the latter are guided by administrative, rather than profit-maximizing considerations and are taken, therefore, as exogenous.

The model adopts the standard assumption that a large number of privately-owned businesses operate in competitive markets, transforming labor and capital inputs into output with constant returns to scale technology that exhibits labor-augmenting technical progress

⁸Again, the presence of the factor $(1 + \eta)(1 + \gamma)$ on the right-hand side of the equation is a direct consequence of removing the deterministic TFP and population growth rates from the capital stock.

and unitary elasticity of substitution between inputs. As is well known, under those conditions the aggregate output of the model economy corresponds to that generated by a single representative firm endowed with a Cobb-Douglas production function:

$$y_t^{pr} = \frac{1}{e^{(1-\theta)\gamma t}} A e^{(1-\theta)z_t} k_t^\theta [e^{\gamma t} h_t^{pr}]^{1-\theta}, \quad (3.7)$$

where y_t^{pr} is the output per working age person produced by private sector firms; θ the proportion of the remuneration to capital services in the private sector value added; and z_t is a stochastic technology level that introduces the fourth wedge implementing the BCA approach incorporated to the methodology proposed by this paper. This technology level shifter corresponds conceptually to the *efficiency wedge* in CKM. The properties of the stochastic process governing its evolution over time will be discussed in section 3.4.2.⁹

The representative firm that stands for the large number of them making decisions in the economy solves, therefore, the following maximization problem:

$$\underset{h_t^{pr}, k_t}{Max} \left[A e^{(1-\theta)z_t} k_t^\theta (h_t^{pr})^{1-\theta} - w_t h_t^{pr} - r_t k_t \right]. \quad (3.8)$$

Notice that in this economy, it is the stand-in household that makes the investment decisions. Absent the intertemporal dimension, the representative firm's problem reduces to a sequence of static, single-period problems.

3.3.3. Public Sector Policies

The allocation of resources by public sector entities is the result of complex social, political, and economic considerations, not aptly captured by the same profit- and utility-

⁹Given that all variables have been detrended, the growth factor e^γ in equation (3.7) is obviously redundant. It was made explicit, however, to emphasize that the model economy is characterized by secular technical progress that the Cobb-Douglas production function permits one to represent as labor augmenting. As shown by Greenwood, Hercowitz, and Krusell (1997), that is the only production function always consistent with balanced growth in the presence of investment-specific, or capital-embodied, technological change, provided the depreciation rate is interpreted as the economic, rather than physical, depreciation rate. The constant economic depreciation rate δ in equations (3.2) and (3.3) implicitly assumes, therefore, a constant growth rate of investment-specific technological progress.

maximizing incentives faced by households and private sector firms. Given the difficulties in modeling explicitly the behavior underlying the economic decisions made by public sector agencies, the variables under their control are assumed to be exogenously determined.

3.3.3.1. *Government Budget Constraint and the Sequester*

Fiscal solvency is imposed in the model economy by imposing the restriction that any change in the government purchases of goods and services (excluding labor services counted in government value added) must be offset by a corresponding change in net revenues. Thus, in the model the government absorption of output exclusively produced by the private sector, denoted ga_t , will be assumed to be equal every period to revenues from all sources minus transfer payments, as indicated by the following government budget constraint:

$$ga_t = \tau_t^h w_t (h_t^{pr} + h_t^{pu}) - w_t h_t^{gc} + \tau^k (r_t - \delta) k_t + s_t^{ge} - \tau_t, \quad (3.9)$$

where h_t^{pu} is equal to $h_t^{gc} + h_t^{ge}$, with h_t^{gc} and h_t^{ge} representing the fraction of time the stand-in household works for government agencies and government-owned enterprises, respectively; s_t^{ge} denotes, for consistency with the NIPA methodology, surpluses (deficits, if negative) transferred by government-owned enterprises; and τ_t stands for lump-sum transfers. In line with the treatment of variables corresponding to physical quantities discussed before, those of the same type in the government budget constraint are measured in units of the consumption good per working age population as well.

For the purposes of the present paper, it will be convenient to interpret the variable ga_t as made up of a systematic, exogenous stochastic component, ega_t , and of a non-systematic, deterministic component, pga_t , as represented by the relationship

$$ga_t = ega_t + pga_t. \quad (3.10)$$

In line with the historical developments described in section 3.2.1, the stochastic component ega_t is meant to capture the ups and downs of the government spending policy historically

followed until the sequestration took place in 2013.

The non-systematic, deterministic component pga_t is meant to capture the “policy regime change” of limited duration (from 2013 to 2021, to be precise) implied by the budget sequestration spending cuts. This policy component of ga_t is a placeholder that in the quantitative implementation of the model will be replaced by the values in the last column of Table 3.2, with the practical effect of shifting down the government absorption of private output relative to the level implied by the exogenous component ega_t .

For consistency with the balanced growth assumption, that exogenous stochastic component is postulated to evolve over time according to a stationary stochastic process with the following autoregressive representation:

$$\ln \frac{ega_t}{y_t^{pr}} = (1 - \rho_{ga}) \ln gy + \rho_{ga} \ln \frac{ega_{t-1}}{y_{t-1}^{pr}} + \sigma_{gy} \varepsilon_t^{gy}, \quad (3.11)$$

where gy and σ_{gy} are scalars; and ε_t^{gy} is a random variable with a standard normal distribution.

3.3.3.2. Public Sector Labor Demand

In line with the pattern of the previous stochastic process, the general government and government enterprises’ demand for labor services is also assumed to be autocorrelated, with the following representation:

$$\ln h_t^{pu} = (1 - \rho_{hpu}) \ln h_{ss}^{pu} + \rho_{hpu} \ln h_{t-1}^{pu} + \sigma_{hpu} \varepsilon_t^{hpu} \quad (3.12)$$

where h_{ss}^{pu} and σ_{hpu} are scalars; and ε_t^{hpu} is a random variable characterized by a standard normal distribution.

3.3.3.3. Government Enterprises Value Added

The value added by government enterprises, va_t^{ge} , which NIPA treats as originated in the private business sector, should grow at the same rate as private sector output along a

balanced growth path. Therefore, it is sensible to assume that the evolution of this variable over time is determined by the following stochastic processes:

$$\ln \frac{va_t^{ge}}{y_t^{pr}} = \ln vy + \sigma_{vy} \varepsilon_t^{ge} \quad (3.13)$$

where vy and σ_{vy} are scalars; and ε_t^{ge} is a random variable characterized by a standard normal distribution.

3.3.3.4. Resource Constraint

For the purpose of subsequent analysis, it is useful to make explicit the resource constraint that results from consolidating the household's budget constraint (3.2) with the government budget constraint (3.9), after taking into account that, for consistency with the NIPA methodology, output in the model economy originates in private sector firms according to (3.7) and in government-owned enterprises according to (3.13), as well as that the operating surpluses of the latter (revenues minus labor costs) are transferred as a lump sum to the households:

$$c_t + (1 + \tau_t^x)x_t = \left[1 + \frac{va_t^{ge}}{y_t^{pr}} - \frac{ga_t}{y_t^{pr}} + \frac{niy_t}{y_t^{pr}} \right] Ae^{(1-\theta)z_t} k_t^\theta (h_t^{pr})^{1-\theta}.$$

3.3.4. Model Calibration

As it should be apparent from the preceding section, the model economy involves a fairly large number of parameters and the attempt of estimating all of them with available statistical tools at an acceptable level of precision is doomed to failure given the limited available data, at most 36 annual observations, from 1977 to 2013, for the aggregate variables of interest. Therefore, it seemed wise to calibrate as many parameter values as possible with the widely accepted quantitative discipline imposed by the requirement that the steady state economic relationships between variables and/or parameters predicted by the model economy should match those prevailing in the actual economy, on average, over fairly long periods of

time.

The parameters of the model economy whose values were set with a calibration approach are listed in Table 3.3. Whenever the calibrated values involved the use of historical averages, they correspond to the period 1997-2007. The observations pertaining to the Great Recession and its aftermath were deliberately excluded, on the grounds that the large changes that many macroeconomic variables experienced during that unusually deep contraction were persistent, but not permanent, and didn't have an everlasting impact, therefore, in the long run trends of the actual economy. The paper will take into account, however, that the permanent increase of the capital income tax rate effective in 2013, mentioned at the end of section 3.2.1, did change the steady state of the economy after it was enacted in 2010.

Missing from that table are the model parameters that can only be inferred from the high frequency movements of the economic variables under their influence, by their nature not identifiable from steady state relationships. Three types of parameters fall in this class: 1) the coefficients of stationary stochastic processes that drop out from the model equations in steady state, 2) parameters controlling intertemporal substitution effects in consumption and labor, the IES and the Frisch elasticity, and 3) parameters whose steady state values depend on these two macroelasticities.

Parameters in the first type of those just listed will be estimated with the techniques discussed in the next section. A different approach is followed, however, for the second type of parameters, the IES and the Frisch elasticity. To avoid the controversies surrounding their empirically relevant values, the paper explored the extent to which the spending cuts prescribed by the budget sequestration were credible for different combinations of values of those macroelasticities, representative of those advocated by some and disputed by others in the literature.

Thus, for the IES, captured by the reciprocal of the parameter σ in the model, the paper will consider the following two values most commonly invoked as empirically relevant in the literature:

- 0.5, and 1.

For the Frisch elasticity, captured by the parameter φ , the paper will consider the following five values:

- 0.5, 1, 1.9, 2.5, and 3.

The first Frisch elasticity value is the median estimate inferred from so-called microeconomic studies, because they estimate that macroelasticity from evidence at the level of individuals or households, rather than from aggregate variables. The value of 1 is suggested by the survey evidence on the response of labor supply to a large wealth shock examined by Kimball and Shapiro (2008). The value of 1.9 has been proposed in an often-cited paper by Hall (2009). The value of 3 has been inferred by Prescott (2004) from a macroeconomic study, in the sense that he drew that as an implication from the behavior of the aggregate labor supply in countries with different labor income tax rates. Finally, the values of 2.5 in between the last two was added to the list for completeness.

Finally, recall that the third type of parameters that could not be calibrated includes those that are implied by steady state relationships that depend, precisely, on the values of the macroelasticities just discussed. That is the case of the parameters κ and β in the utility function.

For example, the Euler equation associated with the intertemporal first order necessary condition for the household's maximization problem described in section 3.3.1 implies the following steady state relationship between the latter parameter and the IES:

$$1 + (1 - \tau^k)(r - \delta) = \frac{(1 + \gamma)^\sigma}{\beta}.$$

Accordingly, the value of β was recalculated for each value of σ , taking into account that the studies by Poterba (1998), Siegel (2002), and Mehra and Prescott (2008) have established with some confidence that the long-run annual real return on capital for the U.S. economy, captured by the factor $(r - \delta)$ in the equation above, is in the order of magnitude of 8%.

A similar procedure was applied to κ , whose dependence on φ is manifested by the intratemporal first order condition of the household's maximization problem.

Table 3.3. Calibration

Parameter	Value	Description
η	0.0126	working-age annual population net growth rate;
γ	0.0078	TFP annual net growth rate;
δ	0.0621	depreciation rate;
i	0.0858	before-tax annual net rate of return on private capital
y_{ss}^{pr}	1.00	private sector output;
x/y^{pr}	0.2121	investment-output ratio;
k/y^{pr}	2.5681	private capital–private sector output ratio;
θ	0.38	private capital income share;
gy	0.0825	fraction of private sector output absorbed by general government;
vy	0.0156	government enterprises value added–private sector output ratio;
σ_{vy}	0.0856	standard deviation of vy ;
niy	0.026	net exports–private sector output ratio;
h_{ss}^{pu}	0.03	fraction of time worked in public sector;
h_{ss}^{pr}	0.21	fraction of time worked in private sector;
τ_{ss}^x	0	investment wedge;
τ_{ss}^h	0.23	labor income tax rate;
τ_{ss}^k	0.35 up to 2011; 0.388 since 2011	capital income tax rate.

3.4. Inferring the Credibility of the Budget Sequestration with a BCA Event-Study Methodology

3.4.1. Overview

This section provides a narrative overview of the “Business Cycle Accounting event-study” methodology developed in the paper, to allow those readers not initially inclined to go over the technical subtleties to then jump directly to the following section, which reports the findings of the paper.

The first step in the implementation of the methodology is the same as in CKM: to represent the model economy in a state-space form, suitable for estimating with maximum likelihood techniques unobserved state variables and the unknown parameters of the stochastic processes controlling the evolution of such variables over time.

As indicated before, the lack of consensus on the values of the IES and the Frisch elasticity suggested the wisdom of not including these two parameters among the list of those to be estimated. Instead, the steps described below were repeated for each of the ten possible combinations of candidate values for those parameters identified in section [3.3.4](#).

The second step of the methodology, also as in CKM, proceeds to estimate the parameters and unobserved state variables with data for the period 1977-2010. The arguments for limiting the evidence to that period, for estimation purposes, will be provided in the more detailed discussion of this step later. It suffices to mention now that an important consideration was the permanent increase of the capital income tax rate effective in 2013 stipulated by 2010 legislation, mentioned at the end of section [3.2.1](#).

In fact, the third step was motivated precisely by that anticipated change in the tax code. Even if enacted in 2010, it seems reasonable to conjecture that households and businesses would have been able to take into account the consequences of that forthcoming tax code change for their decisions only one year later, in 2011. In that case, the macroeconomic variables capturing those decisions were registering in 2013 not only the transitional dynamic effects induced by the anticipated tax policy change, but also the effects of the budget

sequestration spending cuts triggered that same year. Inferences about the credibility of those cuts could be misleading if made with a methodology that fails to isolate the response of macroeconomic variables to each of those two different policy changes.

The third step of the methodology proposed by the paper avoids that potential methodological flaw. Specifically, this third step incorporates the forthcoming higher capital income tax regime in the equilibrium decision rules for 2011 and 2012, with the technique that will be described in due course, taking as given the parameter values estimated in the previous step. This third step also proceeds to calculate the evolution of the state variables up to 2012 implied by the new equilibrium decision rules and associated laws of motion.

The last step, and the one most different from that in CKM, is the critical one for the purposes of this paper. Recall that CKM exploited the state-space representation of the model to recover the wedges that would replicate the data exactly at each point in time and then feed them one at a time in the model economy, in order to establish their marginal effects on the fluctuations of the macroeconomic variables of interest. In this paper, whose goal instead is to assess the credibility of the budget sequestration cuts, what is fed into the model is rather sequences of spending cuts that differ in a certain percentage, roughly ranging from 0% to 100%, from those targeted by the Budget Control Act.

The parameterization of the targeted spending cuts in the manner just sketched gives rise to a range of “credibility scenarios”, each of them capturing the hypothesis that economic agents were making their decisions, when the budget sequestration was launched, as if expecting that only a certain percentage of the targeted spending cuts would, in the end, be enforced. In the empirical implementation of the model, the fully credible spending cuts scenario is captured, therefore, by entering into the decision rules 100% (full size) of the targeted spending cuts. At the opposite extreme, the “zero credibility” scenario is captured by entering in the decision rules 0% of the targeted spending cuts, that is, by feeding in the model decision rules that dismiss the budget sequestration altogether as a credible policy regime change. In between those two polar scenarios, the paper considers a large number of “intermediate credibility” scenarios, indexed by the percentage of the targeted spending

cuts incorporated in the decision rules.

In principle, different configurations of innovations to the wedges will be necessary to replicate the data exactly for each of the spending cuts credibility scenarios considered. The distribution of those shocks, along with that of the estimated unobserved state variables derived in the previous steps, makes it possible to compute the likelihood of the data for alternative spending cuts scenarios and rank them by the value of the corresponding likelihood function. For a given combination of Frisch elasticity and IES values, the credibility scenario that accounts best for the observed performance of macroeconomic variables during the relevant time frame is that for which the likelihood function value is the highest.

Notice that given the possibility, mentioned in section 3.2.1, that economic agents started to incorporate the prospects of the sequester in 2012 rather than, as more widely believed, in 2013, it was necessary to apply sequentially the fourth and last step of the methodology to those two years.

3.4.2. Technical Details

3.4.2.1. *State-Space Representation*

The first step in implementing the adapted BCA approach just outlined is to represent the model in a state-space form, which is accomplished as usual, by specifying transition equations that govern the evolution of state variables over time and measurement equations that define the mapping between the states and the relevant observed data.

In general stochastic equilibrium models as the one in this paper, the link between observables and state variables in the measurement equations is provided by the equilibrium decisions rules which, as already anticipated, this paper computes with the standard practice of approximating the true decision rules with a first order Taylor expansion around the non-stochastic steady state. This ensures a linear mapping between state variables and observables. With the further assumption that the transition from one state to the other is governed by a linear Markov process, the state-state representation of the model economy

of this paper can be formalized by the *transition equation*

$$S_t = TS_{t-1} + Q\omega_t, \quad (3.14)$$

and the *measurement equation*

$$Y_t = DS_{t-1} + C\omega_t. \quad (3.15)$$

In the *transition equation* (3.14), S_t is a 7×1 vector of state variables at the end of period t ; T is a 7×7 matrix; ω_t is a 7×1 vector whose elements are all the exogenous shocks assumed present in the model economy; and Q is a 7×7 matrix whose elements are discussed in detail below.

In the *measurement equation* (3.15), Y_t is the vector of observable variables; D denotes a 7×7 matrix; and C represents a 7×7 matrix.

To see more clearly how the different elements of the model economy presented in the previous sections fit into the state-space representation, it will prove useful to spell out more fully the vectors and matrices involved as follows, starting with those of the transition equation:

$$S_t = [k_{t+1} - k_{ss}, \ln \frac{ega_t}{y_t^{pr}} - \ln gy, \ln h_t^{pu} - \ln h_{ss}^{pu}, z_t - z_{ss}, \frac{\dot{n}i_t}{y_t^{pr}} - niy, \tau_t^h - \tau_{ss}^h, \tau_t^x - \tau_{ss}^x]',$$

where a subindex "ss" identifies the steady state value of the period t variable immediately to the left¹⁰.

¹⁰For consistency with the timing convention adopted in the law of motion of capital (3.3), the capital stock at the end of period t is denoted in the vector S_t as the beginning of period $t + 1$ capital stock, k_{t+1} .

Continue with the matrix T :

$$T = \begin{bmatrix} T_{11} & T_{12} & T_{13} & T_{14} & T_{15} & T_{16} & 0 \\ 0 & \rho_{ga} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_{hpu} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_z & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_{ni} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{\tau h} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_{\tau x} \end{bmatrix},$$

where the first row of this matrix is simply the result of replacing in the law of motion for the private capital stock, (3.3), the equilibrium decision rule for investment, x_t , approximated as a linear function of the end-of-period $t - 1$ state of the economy, that is, of the state variables in S_{t-1} , and of the innovations ω_t hitting the economy in period t . The second and third rows of the matrix T simply replicate the stochastic processes in equations (3.11) and (3.12), respectively. The rest of the rows of this matrix represent the wedges, expressed in terms of ratios to private sector output when appropriate, as stochastic Markovian processes that depend only on their own past. Interactions between these processes were ruled out by assumption, for the same reasons given earlier: the limited data available would have prevented the reliable estimation of the large number of parameters implied by a less parsimonious specification.¹¹

Consider next the vector ω :

$$\omega_t = [\varepsilon_t^{gy}, \varepsilon_t^{hpu}, \varepsilon_t^{ge}, \varepsilon_t^z, \varepsilon_t^{ni}, \varepsilon_t^{\tau h}, \varepsilon_t^{\tau x}]', \quad (3.16)$$

¹¹It is not clear, in any case, that the interactions would be significant, as they are not statistically different from zero in CKM.

where the first three elements corresponds to the innovations identified in equations (3.11), (3.12), and (3.13); and the remaining elements capture the innovations to the four wedges z_t , ni_t , τ_t^h , and τ_t^x . The variance-covariance matrix of this vector, $E[w_t w_t']$, is denoted by Σ and characterized by the following elements:

$$\Sigma = \begin{bmatrix} \Sigma_{11} & 0_{3 \times 4} \\ 0_{4 \times 3} & \Sigma_{22} \end{bmatrix},$$

where Σ_{11} is a 3×3 identity submatrix, and Σ_{22} is a 4×4 submatrix, with diagonal elements equal to 1 and possibly non-zero off-diagonal elements. This specification assumes that the stochastic process for the government absorption of private sector output, characterized by equation (3.11), as well as that for the public sector labor input, characterized by equation (3.12), are orthogonal to all the others, whereas the innovations to the wedges are allowed to be correlated with each other.

Fully spelled out, the 7×7 matrix Q is given by

$$Q = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & Q_{14} & Q_{15} & Q_{16} & Q_{17} \\ \sigma_{gy} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{hpu} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_z & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{ni} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{\tau h} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\tau k} \end{bmatrix},$$

where the elements of the first row are coefficients implied by the linearized equilibrium decision rule for the capital stock and the rest of the elements just capture the standard deviations of all the exogenous stochastic processes in the model.

In the measurement equation, the 7×1 column vector Y_t contains the observable variables:

$$Y_t = [y_t^{pr} - y_{ss}^{pr}, c_t - c_{ss}, x_t - x_{ss}, h_t^{pr} - h_{ss}^{pr}, \ln \frac{ega_t}{y_t^{pr}} - \ln gy, \ln h_t^{pu} - \ln h_{ss}^{pu}, \ln \frac{va_t^{ge}}{y_t^{pr}} - \ln vy]', \quad (3.17)$$

where again a subindex "ss" identifies the steady state value of the corresponding variable.

It is worth to clarify at this point a potential confusion created by the inclusion of the element $\ln \frac{ega_t}{y_t^{pr}} - \ln gy$ in the vector of observables Y_t . Strictly speaking, the variable directly observable in the data is ga_t , not its individual components identified in equation (3.10). However, as that equation makes apparent, in the absence of the temporary policy regime, the systematic stochastic component $\frac{ega_t}{y_t^{pr}}$ would be equal to $\frac{ga_t}{y_t^{pr}}$ and, therefore, observable as well. This equality holds, therefore, between 1997 and 2012, before the budget sequestration was triggered. When it breaks down in 2013, $\frac{ega_t}{y_t^{pr}}$ is no longer observable but it can be inferred from the data and the spending cuts for that year implied by the legislation that enacted the budget sequestration. In particular, in the absence of the spending cuts, the observation $\frac{ga_{2013}}{y_{2013}^{pr}}$ would have been higher by $\frac{pga_{2013}}{y_{2013}^{pr}}$, the amount by which the sequestration would reduce government spending that year, as per the CBO estimate reported in Table 3.2. Thus, $\frac{ega_{2013}}{y_{2013}^{pr}}$ can be inferred from the equality $\frac{ega_{2013}}{y_{2013}^{pr}} = \frac{ga_{2013}}{y_{2013}^{pr}} - \frac{pga_{2013}}{y_{2013}^{pr}}$ implied by equation (3.10).

The 7×7 matrix D can be rewritten as

$$D = \begin{bmatrix} & & & \mathbb{D}_{4 \times 7} & & & \\ 0 & \rho_{ga} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_{hpu} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

where the elements \mathbb{D}_{ij} of the 4×7 submatrix \mathbb{D} consist of the coefficients of the linearized equilibrium decision rules for the endogenous variables in the vector Y_t ; the element ρ_{ga} restates in matrix notation the first term of equation (3.11); and the element ρ_{hpu} restates

that of equation (3.12).

Finally, the 7×7 matrix C is given by

$$C = \begin{bmatrix} & & \mathbb{C}_{4 \times 7} \\ \sigma_{gy} & 0 & 0 \\ 0 & \sigma_{hpu} & 0 & 0_{3 \times 4} \\ 0 & 0 & \sigma_{vy} \end{bmatrix},$$

where the elements \mathbb{C}_{ij} of the 4×7 submatrix \mathbb{C} are obtained from the equilibrium decision rules and the last three rows restate the second term in equations (3.11), (3.12), and (3.13).

3.4.2.2. Estimation of Unknown States and Parameters

The parameters that could not be calibrated exploiting steady state relationships or the findings of other studies had to be inferred statistically from the data. To that effect, the estimation procedure used all the available data for the period 1977-2010, rather than those for the more limited 1977-2007 period adopted as reference for the calibration of the parameters in Table 3.3. The first year in both periods was determined, as indicated earlier, by data availability considerations. The reason to include data for the Great Recession years for the purpose of estimating unknown parameters and state variables is that, by most accounts, several frictions typically present in the economy manifested themselves with particularly intensity during that episode. The observations pertaining to that contraction might contain, therefore, information particularly useful for estimating the parameters of the stochastic processes of the wedges, meant to summarily capture those frictions in the model.

The reason not to use the data after 2010, even if available, was technical in nature: the technique to estimate the not calibrated parameters governing the stochastic processes of the wedges requires stability of the decision rules characterizing the economic agents' choices, a condition that ceases to be satisfied after legislation passed that year enacted, as mentioned before, a permanent increase of 3.8 percentage points in the capital income tax rate that

would take effect three years later. As mentioned in the overview of this section, the paper assumes that economic agents started to fully incorporate this policy regime change in their decisions the following year, in 2011. For consistency, all not calibrated parameters, including those of the stochastic process (3.11) for the government absorption of private sector output, and those of the stochastic process (3.12) for the public sector labor input, were estimated therefore with data for the period 1977-2010.

For this estimation step, the paper took advantage of rather standard maximum likelihood procedures, particularly well suited for implementation when the structural model of the economy can be represented in state-space form. To gain intuition on the nature of those tools, notice that the estimates of the unknown parameters in the matrices T and Q will be influenced by the difference between the data for the variables in the measurement equation and their predicted values implied by the corresponding decision rules, in turn a function of the parameters that need to be estimated. The Kalman filter, included in many econometric software packages, was especially developed to deal with this “circularity” problem. Following standard practice, the initial values of the state variables were set equal to their steady state values whenever necessary to start the algorithm.

It is important to reiterate at this point that, given that the paper doesn’t take a stand on which of the variety of values for the IES and the Frisch elasticity proposed in the literature are empirically relevant, the parameters that are the subject of this section had to be estimated for each of the ten combination of values of those two macroelasticities listed in section 3.3.4.

The resulting sets of estimates of the state variables, autocorrelation coefficients, and relevant variances and covariances were assumed to characterize the joint distribution of the stochastic variables, one of the inputs required to execute the subsequent steps of the modified BCA methodology proposed in this paper described next.

3.4.2.3. *Incorporating the Tax Regime Change*

In order to interpret the dynamics of macroeconomic variables under the effects of the sequester correctly, it is necessary to establish first how that dynamics was altered by the increase of the capital income tax rate repeatedly mentioned before. With all the parameter values fixed by the last step, this could be accomplished with an algorithm capable of simulating the path of the variables of the model during 2011 and 2012, that is, for the years in which the capital income tax change was anticipated, but not effective yet. Juillard (2006) suggested the general principle behind such an algorithm in the context of perturbation methods: treat perfectly anticipated current and future deviations of a policy variable from its steady state value as exogenous deterministic state variables and approximate the decision rules around the steady state with standard perturbation methods.

In the case of the increase in the capital income tax rate under consideration, the algorithm involves adding a deterministic state variable and modifying the state-space representation of the model accordingly, as follows:

$$S_t = \mathfrak{T}S_{t-1} + \mathfrak{Q}\omega_t + M(\tau_{t+1}^k - \tau_{new}^k), \quad (3.18)$$

$$Y_t = \mathfrak{D}S_{t-1} + \mathfrak{C}\omega_t + R(\tau_{t+1}^k - \tau_{new}^k), \quad (3.19)$$

where $t = 2011, 2012$; M and R are matrices of coefficients with dimensions 7×1 ; and τ_{new}^k represents the tax rate on capital income effective since 2013, 0.388, obtained by adding to the capital income tax rate calibrated to the period 1977-2007, 0.35, the surcharge enacted in 2010, 0.038. The matrices \mathfrak{T} , \mathfrak{Q} , \mathfrak{D} , and \mathfrak{C} simply reflect the fact that the elements of those matrices corresponding to decision rules coefficients are different from the corresponding elements in the matrices T , Q , D , and C in the previous step, because they have been computed by linearizing the model equations around the new steady state implied by the permanently higher tax rate. For future reference, keep in mind that it's only the first row of the matrix \mathfrak{Q} that is different from the corresponding row in the matrix Q , because the elements of the other rows correspond to parameters of the exogenous stochastic processes

whose values were kept at those estimated in the previous step.

Notice that the reformulation of the state-space representation expands the state space with the additional variable $[\tau_{t+1}^k - (\tau^k + 0.038)]$, taking into account that investment decisions in period t depend on the after-tax rate of return on period $t + 1$, as the explicit derivation of the Euler equation would make apparent. Thus, when $t = 2011$, τ_{t+1}^k is still at the level of the old capital income tax rate τ^k , 0.35, and the term $[\tau_{t+1}^k - (\tau^k + 0.038)] = -0.038$ effectively adds a perfectly known in advance, non-zero deterministic state variable that, along with the other ones present in the original formulation, determine the linearized equilibrium decision rules. However, when $t = 2012$, those rules cease to be a function of this extra state variable, which drops out of the model because $\tau_{t+1}^k = \tau_{2013}^k = \tau^k + 0.038 = \tau_{new}^k$.

Thus, it would appear that, for the year 2012, the state-state representation of the model simplifies to:

$$S_t = \mathfrak{T}S_{t-1} + \mathfrak{Q}\omega_t, \quad (3.20)$$

$$Y_t = \mathfrak{D}S_{t-1} + \mathfrak{C}\omega_t, \quad (3.21)$$

However, this formulation assumes that households and businesses were not taking seriously the possibility that the sequester would be actually implemented that year. Since the paper doesn't take that assumption for granted, it will be necessary to modify the decision rules for the year 2012 in a way that they capture the opposite assumption, to be subsequently validated or dismissed statistically, that economic agents behaved as if they were certain already that year that the sequester was going to be actually implemented on the next.

It is important to keep in mind that the goal of this step was to determine the effect of the pre-announced tax regime change on the state variables at the end of period 2011 and 2012, whose level will affect the dynamics of macroeconomic variables at the time that those variables started to register as well, perhaps as early as in 2012, the influence of the budget sequestration scheduled for 2013. The next step illustrates precisely the implication of the pre-announced reduction of discretionary spending for the equilibrium decision rules.

3.4.2.4. Incorporating the Budget Sequestration Cuts

Applying to the anticipated spending cuts the same principle behind the algorithm of the preceding section results in the following state-space representation of the model for the years 2012 and 2013:

$$S_t = \mathfrak{T}S_{t-1} + \mathfrak{Q}\omega_t + \mathfrak{M}_t\Delta_{2013}, \quad (3.22)$$

and

$$Y_t = \mathfrak{D}S_{t-1} + \mathfrak{C}\omega_t + \mathfrak{P}_t\Delta_{2013}, \quad (3.23)$$

where $t = 2012, 2013$; Δ_{2013} is a $nx1$ column vector whose elements will capture different spending cuts scenarios discussed in the next section; and \mathfrak{M}_t and \mathfrak{P}_t are conformable matrices, with dimensions $7xn$.

Notice that the matrices \mathfrak{T} , \mathfrak{Q} , \mathfrak{D} , and \mathfrak{C} are the same as those that capture the change in decision rules induced by the capital income tax rate increase because, as argued at the end of section 3.2, the budget sequestration spending cuts were temporary in nature and assumed accordingly not to have any impact on the steady state equilibrium of the economy. Operationally, this means that the steady state value of the spending cuts is zero. Taking into account, as documented in Table 3.3 that the steady-state private sector output has been calibrated to one by the appropriate choice of the technology level in steady state, the deviations of the sequence of the targeted spending cuts from their steady state value are given by the values in the last column of Table 3.2.

The basic idea guiding the methodological steps described in this section is that the issue examined by this paper, the extent to which U.S. households and businesses believed that the budget sequestration would be implemented in the terms originally announced, can be addressed by examining the responses of the endogenous macroeconomic variables in the vector Y_t to sequences of current and future spending cuts that differ in a certain percentage, between 0% and 100%, from those targeted by the budget sequestration.

The paper implements that idea parametrically, by means of two parameters that control the size of the spending cuts fed into the decision rules in order to compute the equilibrium

allocation predicted by the model. The first parameter, ψ_0 , controls the size of the spending cuts for the year 2013, relative to that targeted by the budget sequestration, whereas the second one, ψ_1 , does the same for the spending cuts from 2014 onwards. The spending cuts scenarios briefly discussed in the overview of this methodology are constructed by alternatively assigning to each of the parameters introduced above the values of evenly separated 100 points defined over the interval $[0,1]$. This parametric treatment of the spending cuts made it possible to consider a total of 10,201 credibility scenarios.

A typical scenario will be characterized then by a certain value of ψ_0 , say 0.90, and a certain value of ψ_1 , say 0.07. The interpretation of this particular scenario is that either at the beginning of 2012, or 2013, as the case may be, economic agents were making their decisions as if expecting that the spending cuts actually implemented would be 90% the size of those targeted for 2013, and 7% of the size of those targeted for subsequent years.

The reason to treat the spending cuts for 2013 and subsequent years separately is to allow for scenarios for which the credibility of the spending cuts targeted for the year 2013 is eventually higher than that of the spending cuts scheduled for the rest of the years. It didn't seem reasonable to exclude such scenarios from consideration, given the chronology of events documented in section 3.2.1. These are plausible scenarios for the year 2013, because as the budget sequestration was effectively launched in the first quarter of that year, households and businesses may have correctly perceived that it was too late for that year's legislative agenda to accommodate modifications to the cuts for 2013 confirmed by the American Taxpayer Relief Act enacted on January 1st of that year. The same constraint was less binding for future legislation with the potential to change the cuts targeted for 2014 and beyond.

It is important to emphasize that the distinction between the parameter controlling the size of the spending cuts for 2013 and that controlling the size of the spending cuts for the following years adds flexibility to the credibility scenarios that could be realistically considered, without excluding those characterized by equal values of the parameters ψ_0 and ψ_1 , that is, those for which $\psi_0 = \psi_1$.

Formally, the parameterization of the spending cuts is introduced in the state-space representation of the model with the relationship:

$$\Delta_{2013} = \Psi \left[\frac{0.24}{100}, \frac{0.49}{100}, \frac{0.53}{100}, \frac{0.54}{100}, \frac{0.52}{100}, \frac{0.50}{100}, \frac{0.48}{100}, \frac{0.45}{100}, \frac{0.43}{100} \right]', \quad (3.24)$$

where Ψ is a 9×9 diagonal matrix whose elements are the values of the parameters controlling the size of the spending cuts in each of the scenarios considered, that is:

$$\Psi = \begin{bmatrix} \psi_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \psi_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \psi_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \psi_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \psi_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \psi_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \psi_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \psi_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \psi_1 \end{bmatrix}.$$

Notice that the numerical values in the column vector in equation (3.24) correspond to the size of the targeted spending cuts identified in the last column of Table 3.2. As indicated in the section interpreting the contents of that table, the nine numerical values indicate the absolute magnitude of the sequence of the spending cuts that the budget sequestration would have targeted for an economy with a steady-state output normalized to one. For example, according to the second numerical value in that column vector, the target of the budget sequestration was to reduce the government absorption of private sector output in 2014 by 0.0049, equivalent to 0.49% of the model economy steady-state level of output.

3.4.2.5. Assessing the Credibility of the Budget Sequestration

The last stage of the methodology proposed in this paper is designed to infer the credibility of the targeted budget sequestration of the spending cuts with the metric suggested by maximum likelihood techniques.

For the reasons mentioned in the overview of the methodology, this last step had to be applied sequentially, first to the year 2012 and then to the year 2013, in order to establish statistically from the evidence in which of these two years economic agents more likely started to incorporate in their decisions the possibility that the budget sequestration would be actually implemented. The concrete steps of implementation of this last stage are as follows:

1. Back out the vector (3.16) of realized exogenous shocks that replicate the data exactly for the years 2012 and 2013 for each spending cut scenario and combination of macroelasticities from equation (3.23):

$$\omega_{i,m} = \mathfrak{C}_i^{-1}Y_m - \mathfrak{C}_i^{-1}\mathfrak{D}_iS_{i,m-1} - \mathfrak{C}_i^{-1}\mathfrak{P}_{i,m}\Delta_{j,2013},$$

where the subindex m stands alternatively for the years 2012 and 2013; the subindex i indicates that the elements of the matrix or vector bearing it correspond to those associated with the particular combination i of values of the IES and the Frisch elasticity, out of the ten considered; and the subindex j identifies the particular spending cuts scenario j , out of the 10,201 considered.¹²

2. Calculate the likelihood of the data for years 2012 and 2013 for each spending cuts scenario and combination of macroelasticities. Recall that the state variables and innovations to the wedges have been updated as indicated above, but that all distributional parameters required for the calculation of the likelihood have been kept fixed at the values obtained in the estimation step.¹³

¹²Since there are seven equations (one for each of the seven observables) and seven unknowns (seven exogenous shocks), this step is generally feasible, except in the rare case in which \mathfrak{C}_i happens to be singular.

¹³More specifically, the likelihood of the observables for each of the years 2012 and 2013 can be computed

3. Use the information provided by the likelihood of the data under different combinations of macroelasticity values and spending cuts scenarios to make inferences about the extent to which the fiscal austerity implied by the budget sequestration was credible as a fiscal stabilization tool in the year 2012 and, subsequently, in the year 2013.

3.5. Findings

This section reports the results of applying the last step of the methodology described above, first to the year 2012 and then to the year 2013.

As indicated before, the need to check the likelihood of the vector of observables (3.17) for each of those years separately was suggested by the chronology of events discussed in section 3.2.1, which didn't completely dissipate some ambiguity as to in which of those two years economic agents started to adjust their decisions in response to the spending cuts that the budget sequestration would end up triggering in 2013.

The credibility of the targeted spending cuts as of 2012 is assessed by searching, over all 10,201 scenarios, for the maximum of the likelihood of observables in that particular year, for each of the ten combinations of values for the IES and the Frisch elasticity considered. It turns out that, for all macroelasticities values, the likelihood function attained its maximum value when the two parameters controlling the size of the spending cuts fed into the decision rules are zero. That is, for the scenario identified, for all IES and Frisch elasticity values, by the parameter values $\psi_0 = \psi_1 = 0$.

This result seems to validate the hypothesis that all throughout 2012 households and businesses in the U.S. economy were making decisions as if taking almost for granted that

quite straightforwardly, with the formula [13.4.1] on page 385 in Hamilton (1994), after exploiting the isomorphism between the dynamic system of equations (3.14) and (3.15) and the system $\xi_{t+1} = F\xi_t + G\omega_{t+1}, Y_t = A'x_t + H'\xi_t$, where $\xi_{t+1} \equiv \begin{bmatrix} S_t & \omega_{t+1} \end{bmatrix}'$, $F \equiv \begin{bmatrix} \mathfrak{T} & \mathfrak{Q} \\ 0 & 0 \end{bmatrix}$, $G \equiv \begin{bmatrix} 0 & I \end{bmatrix}'$, I is an identity matrix, $A' \equiv \mathfrak{B}_i$, $x_t \equiv \Delta_t$, and $H' \equiv \begin{bmatrix} \mathfrak{D} & \mathfrak{C} \end{bmatrix}$. To avoid misunderstandings, note that in Hamilton's book the matrix Q denotes the variance-covariance matrix of the state variables, while in the paper, that notation is reserved for the matrix of coefficients of the shocks in the transition equation; and the equation system in Hamilton's book does not incorporate the additional exogenous state variable Δ_t .

lawmakers and policymakers would in the end find a way to prevent the budget sequestration spending cuts from happening.

The paper proceeds then to assess the credibility of the spending cuts as of 2013 with an entirely analogous grid search, but starting the procedure in that year, instead of in 2012.

The likelihood function in this case is maximized, again for all macroelasticity values considered, for the scenario in which economic agents behaved as if expecting that the targeted spending cuts would be fully implemented in 2013, but not at all from 2014 onwards. That is, for the scenario identified by a value of 1 for the parameter ψ_0 and the value of 0 for the parameter ψ_1 . This finding is formally summarized in Table 3.4, which includes also the value of the likelihood function associated with the parameter values just mentioned, $\psi_0 = 1$ and $\psi_1 = 0$.

Table 3.4. Spending Cuts Scenario that Maximizes the Log Likelihood of 2013 Observables

Intertemporal elasticity of substitution ($\sigma = 1$)			
Frisch elasticity (φ)	ψ_0	ψ_1	Likelihood
0.5	1	0	5.9274
1.0	1	0	5.8672
1.9	1	0	5.7935
2.5	1	0	5.7602
3.0	1	0	5.7363
Intertemporal elasticity of substitution ($\sigma = 2$)			
Frisch elasticity (φ)	ψ_0	ψ_1	Likelihood
0.5	1	0	5.8097
1.0	1	0	5.7437
1.9	1	0	5.6525
2.5	1	0	5.6081
3.0	1	0	5.5778

For completeness, Table 3.5 documents the likelihood corresponding to the two extreme credibility scenarios, the full credibility scenario, corresponding to the parameter values $\psi_0 = \psi_1 = 1$, and the complete lack of credibility scenario, captured by the parameter values $\psi_0 = \psi_1 = 0$.

Table 3.5. Log Likelihood of 2013 Observables

Intertemporal elasticity of substitution ($\sigma = 1$)		
	Spending cuts scenario	
Frisch elasticity (φ)	No cuts ($\psi_0 = \psi_1 = 0$)	Full-size statutory cuts ($\psi_0 = \psi_1 = 1$)
0.5	4.4539	5.6191
1.0	4.3866	5.5327
1.9	4.3032	5.4260
2.5	4.2653	5.3776
3.0	4.2383	5.3438
Intertemporal elasticity of substitution ($\sigma = 2$)		
	Spending cuts scenario	
Frisch elasticity (φ)	No cuts ($\psi_0 = \psi_1 = 0$)	Full-size statutory cuts ($\psi_0 = \psi_1 = 1$)
0.5	4.3853	5.6469
1.0	4.3144	5.5648
1.9	4.2140	5.4441
2.5	4.1647	5.3835
3.0	4.1305	5.3413

Overall, the interpretation of these results is that in 2012 economic agents were highly skeptical that the budget sequestration would be triggered in 2013, counting perhaps on legislation then under consideration to at least postpone the prescribed spending cuts indefinitely.

According to the results, that perception changed somewhat in 2013. The failure of the American Taxpayer Relief Act, passed at the very beginning of that year, to postpone the

budget sequestration for more than two months may have played a pivotal role in convincing economic agents that the targeted spending cuts scheduled for that year would be indeed implemented. But that doesn't seem to have been enough to convince households and businesses that the targeted spending cuts for the following years would be actually executed. It turns out that subsequent developments validated those doubts: The Bipartisan Budget Act of 2015 reduced the nominal spending cuts originally scheduled for 2016 and 2017 by \$50 billion and \$20 billion, respectively.

Notice that the combination of macroelasticity values with the highest likelihood in Table 3.4 is that identified by an IES equal to 1 and a Frisch elasticity equal to 0.5. For that reason, the parameters governing the spending cuts scenarios, namely ψ_0 and ψ_1 , are further scrutinized in Appendix C with Bayesian techniques. The results reported therein are largely consistent with those documented in this section.

3.6. Conclusion

Nations confronting structural fiscal imbalances typically attempt to correct them with stabilization programs that significantly alter the existing fiscal policy configuration through steep taxation increases and/or deep government spending reductions.

The variety of outcomes associated with such programs, even those without obvious differences in design or scope, has prompted lively debates in academic and policy forums. Often lost in these exchanges is an important caveat: The outcome of fiscal stabilization programs is not independent of their credibility. As a result, the success or failure of a particular fiscal stabilization program may be attributed to its policy features and design, when, in fact, its credibility may well have been the ultimate determinant of observed outcomes.

The likelihood of such a possibility should be clear from the presence, in virtually every actual economy, of the time-inconsistency mechanism uncovered by Kydland and Prescott (1977) and Calvo (1978): Forward-looking households and businesses will not make the same decisions in the present if they expect an announced fiscal stabilization program to be actually followed through, as they would if they anticipate that it will be repudiated.

Put differently, two plans with exactly the same design can produce different outcomes if one is fully credible and the other is not. An assessment of those programs' credibility could go a long way toward settling policy debates prompted by their different outcomes, particularly when the programs are of similar contours.

The scarcity of formal attempts to establish the credibility of fiscal policy stabilization experiences is surprising. Motivated by the need to address that apparent void in the literature, this paper formally assessed the credibility of a recent fiscal stabilization attempt: that initiated by the budget sequestration spending cuts triggered in the U.S. by the Budget Control Act of 2011.

In the absence of readily available methodologies to make such an assessment, the paper developed a novel one, merging an "event-study" approach typically used to study the effects of fiscal shocks and a "business cycle accounting" approach originally developed to address economic fluctuations questions.

The resulting blended methodology made it possible to assess the credibility of the spending cuts targeted by the Budget Control Act with a well-established statistical metric, the maximum likelihood criterion.

An important step for the application of that metric was the construction of a rather comprehensive set of "spending cuts scenarios". Leaving some minor details aside, each of the scenarios is basically characterized by forward-looking economic agents which, in the abstraction of the model, make their economic decisions, starting either in 2012 or 2013, in expectation that actually implemented spending cuts will be a fraction of those targeted by the Budget Control Act.

The scenarios "device", as used in the paper, made it possible to exploit the wedges introduced by the BCA approach to capture, in an expedient fashion, the presence in the economy of not explicitly modeled frictions. In order to replicate the data exactly for each of the IES and Frisch elasticity values considered, the configuration of the innovations to those wedges must change across "credibility scenarios". The more likely the configurations of the resulting innovations, the higher the value of the likelihood function implied by the

state-space representation of the model economy.

That intuition is formally captured by ranking the credibility scenarios, for each of the 10 possible combinations of macroelasticities considered, by the value of the likelihood function induced by the model economy's state-space representation.

By that standard, the paper's finding can be succinctly summarized as stating that the budget sequestration spending cuts had little to no credibility, regardless of values assigned to the IES and Frisch elasticity. The application of Bayesian inference techniques to the pair of values of the two macroelasticities that delivered the highest likelihood function value also favors low credibility scenarios.

Confidence in the finding, as inferred with the maximum likelihood metric or with a Bayesian metric for a special case, is subject to the limitations of data scarcity inherent in the event-study perspective incorporated into the methodology of the paper. Such a perspective was dictated not only by the nature of the relevant evidence, but also by the fact that the government spending austerity program, whose credibility this paper set to assess, was still unfolding at the time of this writing.

On the other hand, lack of credibility of the targeted budget sequestration spending cuts is the assessment that ought to be expected, limited information notwithstanding, from a methodology capable of anticipating that those cuts would be subsequently watered down. That is exactly what happened: The Bipartisan Budget Act of 2015 reduced the nominal spending cuts originally scheduled for 2016 and 2017 by \$50 billion and \$20 billion, respectively.

That development suggests that the methodology proposed in this paper could be of value to scholars, policymakers, and even private sector advisors interested in extracting early hints about the likely future course of fiscal stabilization programs from the observed performance of macroeconomic variables around the time of their announcement and/or actual implementation.

In any case, as noted at the beginning of this conclusion, the lack of credibility of the budget sequestration detected by this paper should inform future research seeking to evaluate

its ultimate outcome. It could be potentially misleading to extrapolate the policy lessons of the budget sequestration to other fiscal stabilization programs, absent taking into account the finding of this paper. Similar, but credible fiscal austerity programs, may be able to deliver qualitatively and/or quantitatively different outcomes.

That observation applies, of course, to all fiscal stabilization programs. Therein lies the importance of formally assessing the credibility of as many past stabilization programs as possible, as well as of those to come. The paper developed a methodology to assess the credibility of one of those programs. Properly adapted and expanded, that methodology could prove useful to systematically assess the credibility of many other fiscal stabilization programs, the impact of that credibility on macroeconomic outcomes and, ultimately, on the success of the corresponding programs in eliminating structural fiscal imbalances.

APPENDIX A

Appendix of Chapter 1

A.1. Equilibrium Conditions in Stationary Form

In Appendix A.1, we transform the equilibrium conditions described in section 1.2 into their corresponding stationary forms. First, we define the stochastic trend of output as $X_t^Y = A_t^{\alpha/(1-\alpha)} X_t$, and the stochastic trend of capital stock as $X_t^K = A_t^{1/(1-\alpha)} X_t$. Then, we detrend the non-stationary variables as follows:

$$\tilde{Y}_t = \frac{Y_t}{X_t^Y}; \quad \tilde{K}_t = \frac{K_t}{X_{t-1}^K}; \quad \tilde{C}_t = \frac{C_t}{X_t^Y}; \quad \tilde{I}_t = \frac{I_t}{X_t^K}; \quad \tilde{G}_t = \frac{G_t}{X_t^G};$$

$$\tilde{\Lambda}_t = \frac{\Lambda_t}{(X_t^Y)^{-\gamma}}; \quad \tilde{Q}_t = \frac{Q_t}{A_t}; \quad \tilde{P}_t = \frac{P_t}{X_t^Y}; \quad \tilde{V}_t = \frac{V_t}{X_t^Y}.$$

Note that the growth rates of output and capital are given by

$$\mu_t^y = \frac{X_t^Y}{X_{t-1}^Y} = \frac{X_t}{X_{t-1}} \left(\frac{A_t}{A_{t-1}} \right)^{\alpha/(\alpha-1)} = \mu_t^x (\mu_t^a)^{\alpha/(\alpha-1)}; \quad (\text{A.1})$$

$$\mu_t^k = \frac{X_t^K}{X_{t-1}^K} = \frac{X_t}{X_{t-1}} \left(\frac{A_t}{A_{t-1}} \right)^{1/(\alpha-1)} = \mu_t^x (\mu_t^a)^{1/(\alpha-1)}. \quad (\text{A.2})$$

In addition, given our assumption that

$$X_t^G = (X_{t-1}^G)^{\rho_{xg}} (X_{t-1}^Y)^{1-\rho_{xg}},$$

we define

$$X_t^g = \frac{X_t^G}{X_t^Y}.$$

Therefore, we can rewrite the equilibrium conditions in the following stationary form:

$$\widetilde{K_{t+1}} = [1 - \delta(u_t)] \frac{\widetilde{K_t}}{\mu_t^k} + e^{z_t^I} \widetilde{I_t} \left[1 - \Phi \left(\frac{\widetilde{I_t \mu_t^k}}{\widetilde{I_{t-1}}} \right) \right], \quad (\text{A.3})$$

$$\widetilde{C_t} + \widetilde{I_t} + \widetilde{G_t x_t^g} = \widetilde{Y_t}. \quad (\text{A.4})$$

$$\widetilde{Y_t} = e^{z_t} \left(\frac{u_t \widetilde{K_t}}{\mu_t^k} \right)^\alpha (H_t)^{1-\alpha}, \quad (\text{A.5})$$

$$\begin{aligned} \widetilde{\Lambda_t} = & e^{\xi_t} \left(\widetilde{C_t} - b_c \frac{\widetilde{C_{t-1}}}{\mu_t^y} \right)^{-\gamma} (\tau + b_l H_{t-1} - H_t)^{\chi(1-\gamma)} \\ & - \beta b_c E_t \left\{ e^{\xi_{t+1}} \left(\widetilde{C_{t+1} \mu_{t+1}^y} - b_c \widetilde{C_t} \right)^{-\gamma} (\tau + b_l H_t - H_{t+1})^{\chi(1-\gamma)} \right\}; \end{aligned} \quad (\text{A.6})$$

$$\begin{aligned} \widetilde{\Lambda_t} e^{z_t} (1 - \alpha) \left(\frac{u_t \widetilde{K_t}}{\mu_t^k} \right)^\alpha (H_t)^{-\alpha} = & \chi e^{\xi_t} \left(\widetilde{C_t} - b_c \frac{\widetilde{C_{t-1}}}{\mu_t^y} \right)^{1-\gamma} (\tau + b_l H_{t-1} - H_t)^{\chi(1-\gamma)-1} \\ & - \chi \beta b_l E_t \left\{ e^{\xi_{t+1}} \left(\widetilde{C_{t+1} \mu_{t+1}^y} - b_c \widetilde{C_t} \right)^{1-\gamma} (\tau + b_l H_t - H_{t+1})^{\chi(1-\gamma)-1} \right\}; \end{aligned} \quad (\text{A.7})$$

$$\begin{aligned} \widetilde{\Lambda_t} = & \widetilde{\Lambda_t} \widetilde{Q_t} e^{z_t^I} \left[1 - \Phi \left(\frac{\widetilde{I_t \mu_t^k}}{\widetilde{I_{t-1}}} \right) - \left(\frac{\widetilde{I_t \mu_t^k}}{\widetilde{I_{t-1}}} \right) \Phi' \left(\frac{\widetilde{I_t \mu_t^k}}{\widetilde{I_{t-1}}} \right) \right] \\ & + \beta E_t \left\{ \widetilde{\Lambda_{t+1}} \widetilde{Q_{t+1}} \mu_{t+1}^a (\mu_{t+1}^y)^{-\gamma} e^{z_{t+1}^I} \left(\frac{\widetilde{I_{t+1} \mu_{t+1}^k}}{\widetilde{I_t}} \right)^2 \Phi' \left(\frac{\widetilde{I_{t+1} \mu_{t+1}^k}}{\widetilde{I_t}} \right) \right\}; \end{aligned} \quad (\text{A.8})$$

$$\begin{aligned} \widetilde{\Lambda_t} \widetilde{Q_t} = & \beta E_t \left\{ \widetilde{\Lambda_{t+1}} \mu_{t+1}^a (\mu_{t+1}^y)^{-\gamma} \left[e^{z_{t+1}} \alpha u_{t+1}^\alpha \left(\frac{\widetilde{K_{t+1}}}{\mu_{t+1}^k} \right)^{\alpha-1} (H_{t+1})^{1-\alpha} + \widetilde{Q_{t+1}} (1 - \delta(u_{t+1})) \right] \right\}; \end{aligned} \quad (\text{A.9})$$

$$e^{z_t} \alpha u_t \left(\frac{u_t \widetilde{K}_t}{\mu_t^k} \right)^{\alpha-1} (H_t)^{1-\alpha} = \widetilde{Q}_t \delta' (u_t) ; \quad (\text{A.10})$$

$$\widetilde{P}_t = \alpha \widetilde{Y}_t - \widetilde{I}_t ; \quad (\text{A.11})$$

$$R_t^{rf} = \frac{1}{\beta} E_t \left[\frac{\widetilde{\Lambda}_t}{\widetilde{\Lambda}_{t+1}} (\mu_{t+1}^y)^\gamma \right] ; \quad (\text{A.12})$$

$$\widetilde{V}_t = \beta E_t \left[\left(\frac{\widetilde{\Lambda}_{t+1}}{\widetilde{\Lambda}_t} \right) (\mu_{t+1}^y)^{1-\gamma} \left(\widetilde{V}_{t+1} + \alpha \widetilde{Y}_{t+2} - \widetilde{I}_{t+2} \right) \right] ; \quad (\text{A.13})$$

$$x_t^g = (x_{t-1}^g)^{\rho_{xg}} (\mu_t^y)^{-1} . \quad (\text{A.14})$$

Therefore, the complete set of equilibrium conditions in stationary form are given by equations (A.1) - (A.14).

A.2. Analytical Solution to Steady State Equilibrium

Throughout Appendix A.2, for any variable J_t , we drop the time index t and let J_{ss} denote its steady state. Given the parameters and the calibrated values of a subset of the modeled variables, it is straightforward to show that $\mu_{ss}^k = \mu_i^i$. Therefore, equation (A.6) evaluated at the steady state implies that

$$\widetilde{Q}_{ss} = 1. \quad (\text{A.15})$$

Also, from equation (A.10), we observe that

$$\frac{\widetilde{K}_{ss}}{\mu_{ss}^k} = \left(\frac{\delta_1}{\alpha} \right)^{1/(\alpha-1)} H_{ss}. \quad (\text{A.16})$$

Plugging equation (A.15) and (A.16) into (A.9), we have

$$\left(\frac{1}{\beta} \right) \frac{(\mu_{ss}^y)^\gamma}{\mu_{ss}^a} = \delta_1 + 1 - \delta_0. \quad (\text{A.17})$$

So as to ensure that the steady state value of capital utilization rate, u_{ss} , is unity, δ_1 is implicitly determined by equations (A.8) - (A.10).

Rewrite equation (A.16) as

$$\frac{\widetilde{K}_{ss}}{\mu_{ss}^k} (H_{ss})^{-1} = \left(\frac{\delta_1}{\alpha} \right)^{1/(\alpha-1)}. \quad (\text{A.18})$$

Then, plugging equation (A.18) into (A.7) yields

$$\begin{aligned} \widetilde{\Lambda}_{ss} (1 - \alpha) \left(\frac{\delta_1}{\alpha} \right)^{\alpha/(\alpha-1)} &= \chi \left(\widetilde{C}_{ss} - b_c \widetilde{\frac{C_{ss}}{\mu_{ss}^y}} \right)^{1-\gamma} (\tau + b_l H_{ss} - H_{ss})^{\chi(1-\gamma)-1} \\ &\quad - \chi \beta \left(\widetilde{C}_{ss} \mu_{ss}^y - b_c \widetilde{C}_{ss} \right)^{1-\gamma} (\tau + b_l H_{ss} - H_{ss})^{\chi(1-\gamma)-1}. \end{aligned}$$

Let $M = (1 - \alpha) \left(\frac{\delta_1}{\alpha} \right)^{\alpha/(\alpha-1)}$, and rewrite the above equation as

$$\begin{aligned} \widetilde{\Lambda}_{ss} M &= \chi \left(\widetilde{C}_{ss} - b_c \frac{\widetilde{C}_{ss}^y}{\mu_{ss}^y} \right)^{1-\gamma} (\tau + b_l H_{ss} - H_{ss})^{\chi(1-\gamma)-1} \\ &\quad - \chi \beta b_l \left(\widetilde{C}_{ss} \mu_{ss}^y - b_c \widetilde{C}_{ss} \right)^{1-\gamma} (\tau + b_l H_{ss} - H_{ss})^{\chi(1-\gamma)-1}. \end{aligned} \quad (\text{A.19})$$

In the steady state, equation (A.6) suggests that

$$\begin{aligned} \widetilde{\Lambda}_{ss} &= (\mu_{ss}^y)^\gamma \left(\mu_{ss}^y \widetilde{C}_{ss} - b_c \widetilde{C}_{ss} \right)^{-\gamma} (\tau + (b_l - 1) H_{ss})^{\chi(1-\gamma)} \\ &\quad - \beta b_c \left(\widetilde{C}_{ss} \mu_{ss}^y - b_c \widetilde{C}_{ss} \right)^{-\gamma} (\tau + (b_l - 1) H_{ss})^{\chi(1-\gamma)}. \end{aligned} \quad (\text{A.20})$$

Combining equations (A.19) and (A.20) we have

$$\begin{aligned} &\chi (\mu_{ss}^y)^{\gamma-1} \left(\mu_{ss}^y \widetilde{C}_{ss} - b_c \widetilde{C}_{ss} \right)^{1-\gamma} (\tau + (b_l - 1) H_{ss})^{\chi(1-\gamma)-1} - \chi \beta b_l \left(\widetilde{C}_{ss} \mu_{ss}^y - b_c \widetilde{C}_{ss} \right)^{1-\gamma} (\tau + (b_l - 1) H_{ss})^{\chi(1-\gamma)-1} \\ &= M (\mu_{ss}^y)^\gamma \left(\mu_{ss}^y \widetilde{C}_{ss} - b_c \widetilde{C}_{ss} \right)^{-\gamma} (\tau + (b_l - 1) H_{ss})^{\chi(1-\gamma)} - \beta b_c \left(\widetilde{C}_{ss} \mu_{ss}^y - b_c \widetilde{C}_{ss} \right)^{-\gamma} (\tau + (b_l - 1) H_{ss})^{\chi(1-\gamma)}. \end{aligned} \quad (\text{A.21})$$

Dividing both sides of equation (A.21) by $\left(\widetilde{C}_{ss} \mu_{ss}^y - b_c \widetilde{C}_{ss} \right)^{-\gamma} (\tau + (b_l - 1) H_{ss})^{\chi(1-\gamma)-1}$ yields

$$\chi (\mu_{ss}^y - b_c) \left[(\mu_{ss}^y)^{\gamma-1} - \beta b_l \right] \widetilde{C}_{ss} = M [(\mu_{ss}^y)^\gamma - \beta b_c] [\tau + (b_l - 1) H_{ss}],$$

by which we can solve for \widetilde{C}_{ss} using H_{ss} :

$$\widetilde{C}_{ss} = \frac{M [(\mu_{ss}^y)^\gamma - \beta b_c] [\tau + (b_l - 1) H_{ss}]}{\chi (\mu_{ss}^y - b_c) \left[(\mu_{ss}^y)^{\gamma-1} - \beta b_l \right]}. \quad (\text{A.22})$$

Using equations (A.16) and (A.3), we are able to solve for \widetilde{I}_{ss} (in terms of H_{ss}):

$$\widetilde{I}_{ss} = H_{ss} \left(\mu_{ss}^k + \delta_0 - 1 \right) \left(\frac{\delta_1}{\alpha} \right)^{1/(\alpha-1)}. \quad (\text{A.23})$$

In addition, equation (A.5) implies that

$$\widetilde{Y}_{ss} = H_{ss} \left(\frac{\delta_1}{\alpha} \right)^{\alpha/(\alpha-1)} \quad (\text{A.24})$$

Since we calibrate the steady state share of government expenditures in output gy , it suggests that

$$gy\widetilde{Y}_{ss} = \widetilde{G}_{ss}x_{ss}^g. \quad (\text{A.25})$$

Given the solution to \widetilde{C}_{ss} , \widetilde{I}_{ss} and \widetilde{Y}_{ss} , plugging equation (A.22) - (A.25) into (A.4) yields

$$\begin{aligned} & \left\{ (1 - gy) \left(\frac{\delta_1}{\alpha} \right)^{\alpha/(\alpha-1)} - (\mu_{ss}^k + \delta_0 - 1) \left(\frac{\delta_1}{\alpha} \right)^{1/(\alpha-1)} - \frac{M[(\mu_{ss}^y)^\gamma - \beta b_c][(b_l - 1)H_{ss}]}{\chi(\mu_{ss}^y - b_c)[(\mu_{ss}^y)^{\gamma-1} - \beta b_l]} \right\} \\ & = \frac{M[(\mu_{ss}^y)^\gamma - \beta b_c]\tau}{\chi(\mu_{ss}^y - b_c)[(\mu_{ss}^y)^{\gamma-1} - \beta b_l]} \end{aligned} \quad (\text{A.26})$$

which allows us to solve for H_{ss} , which is given by

$$H_{ss} = \left\{ (1 - gy) \left(\frac{\delta_1}{\alpha} \right)^{\alpha/(\alpha-1)} - \frac{M[(\mu_{ss}^y)^\gamma - \beta b_c][(b_l - 1)]}{\chi(\mu_{ss}^y - b_c)[(\mu_{ss}^y)^{\gamma-1} - \beta b_l]} \right\}^{-1} \frac{M[(\mu_{ss}^y)^\gamma - \beta b_c]\tau}{\chi(\mu_{ss}^y - b_c)[(\mu_{ss}^y)^{\gamma-1} - \beta b_l]}. \quad (\text{A.27})$$

Plugging the solution to H_{ss} in equation (A.16), (A.22), (A.23) and (A.24), we can solve for to \widetilde{K}_{ss} , \widetilde{C}_{ss} , \widetilde{I}_{ss} and \widetilde{Y}_{ss} accordingly. In addition, equation (A.20), combined with \widetilde{C}_{ss} and H_{ss} solves $\widetilde{\Lambda}_{ss}$. Furthermore, given $\widetilde{\Lambda}_{ss}$, \widetilde{I}_{ss} and \widetilde{Y}_{ss} , equations (A.11) - (A.13), evaluated at the steady state, would allow us to solve for \widetilde{P}_{ss} , R_{ss}^{rf} and \widetilde{V}_{ss} .

Till now, the only remaining task is to solve for \widetilde{G}_{ss} and x_{ss}^g . From equation (A.14), it is straightforward to show that

$$x_{ss}^g = (\mu_{ss}^y)^{1/(\rho x_g - 1)}. \quad (\text{A.28})$$

Finally, $\widetilde{G_{ss}}$ is solved by using equation (A.25) and the solution to x_{ss}^g and $\widetilde{Y_{ss}}$.

A.3. Data Construction

The quarterly data on output growth, investment growth, consumption to output ratio, government expenditures to output ratio, total market values to output ratio, hours and risk-free rate is constructed using the following series:

(1) Nominal Gross Domestic Product, downloaded from BEA (www.bea.gov), National Income and Product Accounts Table 1.1.5 (Quarterly), line 1, billions of dollars seasonally adjusted at annual rate;

(2) Real Gross Domestic Product, downloaded from BEA (www.bea.gov), National Income and Product Accounts Table 1.1.6 (Quarterly), line 1, billions of chained 2009 dollars seasonally adjusted at annual rate;

(3) Nominal Nonresidential Fix Investment, downloaded from BEA (www.bea.gov), National Income and Product Accounts Table 1.1.5 (Quarterly), line 9, billions of dollars seasonally adjusted at annual rate;

(4) Nominal Residential Fix Investment, downloaded from BEA (www.bea.gov), National Income and Product Accounts Table 1.1.5 (Quarterly), line 13, billions of dollars seasonally adjusted at annual rate;

(5) Implicit Deflator for Fixed Investment, downloaded from BEA (www.bea.gov), National Income and Product Accounts Table 1.1.9 (Quarterly), line 8, seasonally adjusted;

(6) Nominal Personal Consumption on Nondurable Goods, downloaded from BEA (www.bea.gov), National Income and Product Accounts Table 1.1.5 (Quarterly), line 5, billions of dollars seasonally adjusted at annual rate;

(7) Nominal Personal Consumption on Services, downloaded from BEA (www.bea.gov), National Income and Product Accounts Table 1.1.5 (Quarterly), line 6, billions of dollars seasonally adjusted at annual rate;

(8) Implicit Deflator for Personal Consumption Expenditure, downloaded from BEA (www.bea.gov), National Income and Product Accounts Table 1.1.9 (Quarterly), line 2, seasonally adjusted;

(9) Nominal Government Gross Investment, downloaded from Bureau of Economic Analysis (www.bea.gov), National Income and Product Accounts Table 3.9.5 (Quarterly), line 3, billions of dollars, seasonally adjusted at annual rate;

(10) Nominal Government Consumption Expenditure, downloaded from BEA (www.bea.gov), National Income and Product Accounts Table 3.9.5 (Quarterly), line 2, billions of dollars, seasonally adjusted at annual rate;

(11) Nonfarm Business Hours Worked, BLS label PRS85006033, downloaded from FRED (research.stlouisfed.org), index 2009=100, seasonally adjusted;

(12) Civilian Non-institutional Population over 16, BLS label LNU00000000Q, downloaded from BLS (www.bls.gov);

(13) 3-Month Treasury Bill Secondary Market Rate (Monthly), downloaded from FRED (research.stlouisfed.org), not seasonally adjusted;

(14) Total Market Values, CRSP data with all the stocks traded on NYSE, AMEX and NASDAQ included, downloaded from WRDS (wrds-web.wharton.upenn.edu/wrds/), thousands of dollars.

The construction of data on Y_t , I_t , A_t , C_t , G_t , M_t , and H_t is straightforward:

$$\text{GDP Deflator } (GDef_t) = (1)/(2);$$

$$\text{Real Per Capita GDP } (Y_t) = (2)/(12);$$

$$\text{Real Per Capita Investment } (I_t) = [(3) + (4)] / (12) / Gdef;$$

$$\text{Relative Price of Investment } (A_t) = (5)/(8);$$

$$\text{Real Per Capita Consumption } (C_t) = [(6) + (7)] / (12) / Gdef;$$

$$\text{Real Per Capita Government Expenditure } (G_t) = [(9) + (10)] / (12) / Gdef;$$

$$\text{Per Capita Hours Worked } (H_t) = (5)/(8).$$

In this step, note that Y_t , I_t , A_t , C_t and G_t are scaled by 1,000,000 simply because they are measured in billions of dollars whereas M_t is measured in thousands of dollars.

The construction of the gross real risk-free rate takes a few more steps. First, monthly data on 3-month treasury bill rate is converted into quarterly by taking the average of the three observations in each quarter. Second, given that 3-month treasury bill rate is the annualized return measured by percentage points, the data on the quarterly series is divided by 400. In addition, we construct the real risk-free rate by making the assumption of perfect foresight. Based on Fisher's equation, we subtract expected inflation, $\log \left(\frac{GDef_{t+1}}{GDef_t} \right)$, and add 1 to obtain the gross real risk-free rate.

A.4. Additional Tables

Table A.1. Unconditional Variance Decomposition : Specification 2

	Shock	g^Y	g^{AI}	$\log\left(\frac{C}{Y}\right)$	$\log\left(\frac{G}{Y}\right)$	$\log\left(\frac{V}{Y}\right)$	H	R^{rf}
Persistent	$\epsilon^{x,P}$	/	/	/	/	/	/	/
	$\epsilon^{a,P}$	0.4587	0.3894	0.2998	0.4440	0.7267	0.6304	0.1084
	$\epsilon^{z,P}$	0.0067	0.0251	0.0053	0.0040	0.0016	0.0157	0.1926
Sum		0.4654	0.4145	0.3051	0.4480	0.7283	0.6461	0.3031
Transitory	$\epsilon^{x,T}$	0.0507	0.0117	0.0005	0.0017	0.0004	0.0006	0.0113
	$\epsilon^{a,T}$	0.4614	0.5313	0.1675	0.3435	0.2698	0.3275	0.1044
	$\epsilon^{z,T}$	0.0028	0.0111	0.0000	0.0001	0.0008	0.0000	0.5101
	ϵ^ξ	0.0004	0.0001	0.5267	0.1996	0.0004	0.0255	0.0013
	ϵ^{z^I}	0.0070	0.0297	0.0001	0.0006	0.0002	0.0001	0.0154
	ϵ^g	0.0123	0.0015	0.0000	0.0063	0.0001	0.0000	0.0566
Sum		0.5346	0.5854	0.6948	0.5518	0.2717	0.3537	0.6991

Table A.2. Unconditional Variance Decomposition : Specification 3

	Shock	g^Y	g^{AI}	$\log\left(\frac{C}{Y}\right)$	$\log\left(\frac{G}{Y}\right)$	$\log\left(\frac{V}{Y}\right)$	H	R^{rf}
Persistent	$\epsilon^{x,P}$	0.5854	0.6343	0.5125	0.1277	0.8770	0.5601	0.5418
	$\epsilon^{a,P}$	/	/	/	/	/	/	/
	$\epsilon^{z,P}$	0.0031	0.0050	0.0048	0.0055	0.0009	0.0301	0.0668
Sum		0.5885	0.6393	0.5173	0.1332	0.8779	0.5902	0.6086
Transitory	$\epsilon^{x,T}$	0.3681	0.2856	0.1511	0.0720	0.0952	0.1247	0.1743
	$\epsilon^{a,T}$	0.0180	0.0221	0.0170	0.0066	0.0095	0.0160	0.0070
	$\epsilon^{z,T}$	0.0069	0.0148	0.0010	0.0019	0.0009	0.0011	0.1822
	ϵ^ξ	0.0013	0.0022	0.0026	0.0000	0.0002	0.0001	0.0149
	ϵ^{z^I}	0.0142	0.0358	0.0482	0.0322	0.0164	0.1566	0.0087
	ϵ^g	0.0023	0.0001	0.2628	0.7541	0.0000	0.1113	0.0042
Sum		0.4116	0.3606	0.4827	0.8668	0.1222	0.4098	0.3913

Table A.3. Unconditional Variance Decomposition : Specification 4

	Shock	g^Y	g^{AI}	$\log\left(\frac{C}{Y}\right)$	$\log\left(\frac{G}{Y}\right)$	$\log\left(\frac{V}{Y}\right)$	H	R^{rf}
Persistent	$\epsilon^{x,P}$	/	/	/	/	/	/	/
	$\epsilon^{a,P}$	/	/	/	/	/	/	/
	$\epsilon^{z,P}$	0.3723	0.0633	0.0524	0.5616	0.0095	0.0829	0.1035
Sum		/	/	/	/	/	/	/
Transitory	$\epsilon^{x,T}$	0.2770	0.0769	0.0474	0.0061	0.0298	0.0077	0.1243
	$\epsilon^{a,T}$	0.0873	0.4429	0.7889	0.1937	0.9332	0.1153	0.1746
	$\epsilon^{z,T}$	0.1049	0.0115	0.0012	0.0037	0.0002	0.0008	0.3958
	ϵ^ξ	0.0821	0.0216	0.0212	0.1935	0.0142	0.7667	0.1048
	ϵ^{z^I}	0.0647	0.3828	0.0867	0.0238	0.0129	0.0261	0.0624
	ϵ^g	0.0116	0.0011	0.0022	0.0176	0.0002	0.0005	0.0346
Sum		0.6276	0.9368	0.9476	0.4384	0.9905	0.9171	0.8965

Table A.4. Bayesian Estimation of Structural Parameters : Specification 2

Parameter	Prior			Posterior				
	Distribution	Mean	Std.	Mean	Median	Std.	Percentile 10%	Percentile 90%
γ	Gamma	2	1	1.3758	1.3767	0.1106	1.3542	1.3956
κ	Gamma	4	2	7.0746	7.0769	0.0367	7.0556	7.0889
b_c	Beta	0.5	0.2	0.7377	0.7376	0.0151	0.7346	0.7417
b_l	Beta	0.5	0.2	0.8931	0.8933	0.0235	0.8895	0.8960
χ	Gamma	4	2	4.5771	4.5756	0.1642	4.5576	4.5993
δ_2	Gamma	0.1	0.05	0.1259	0.1260	0.0012	0.1254	0.1263
$\phi_1^{x,T}$	Beta	0.6	0.3	0.1543	0.1523	0.0248	0.1443	0.1660
$\phi_2^{x,T}$	Beta	0.2	0.1	0.1309	0.1308	0.0062	0.1286	0.1332
$\phi_1^{a,T}$	Beta	0.6	0.3	0.6906	0.6915	0.0048	0.6838	0.6974
$\phi_2^{a,T}$	Beta	0.2	0.1	0.2504	0.2506	0.0060	0.2461	0.2549
$\phi_1^{z,T}$	Beta	0.6	0.3	0.3099	0.3116	0.0106	0.3032	0.3154
$\phi_2^{z,T}$	Beta	0.2	0.1	0.1836	0.1838	0.0086	0.1814	0.1851
ϕ_1^{ξ}	Beta	0.6	0.3	0.6549	0.6546	0.0222	0.6532	0.6568
ϕ_2^{ξ}	Beta	0.2	0.1	0.3448	0.3452	0.0090	0.3430	0.3466
$\phi_1^{z^I}$	Beta	0.6	0.3	0.0870	0.0874	0.0339	0.0828	0.0908
$\phi_2^{z^I}$	Beta	0.2	0.1	0.2339	0.2343	0.0051	0.2314	0.2358
ϕ_1^g	Beta	0.6	0.3	0.4955	0.4958	0.0096	0.4921	0.4985
ϕ_2^g	Beta	0.2	0.1	0.3000	0.2995	0.0050	0.2974	0.3031
ρ^{xg}	Beta	0.7	0.3	0.5745	0.5740	0.0071	0.5684	0.5817
$\sigma_{x,P}$	/	/	/	/	/	/	/	/
$\sigma_{x,T}$	I-G	0.1	Inf.	0.0202	0.0201	0.0024	0.0193	0.0213
$\sigma_{a,P}$	I-G	0.02	Inf.	0.0135	0.0134	0.0015	0.0285	0.0322
$\sigma_{a,T}$	I-G	0.1	Inf.	0.0306	0.0306	0.0015	0.0285	0.0322
$\sigma_{z,P}$	I-G	0.02	Inf.	0.0247	0.0247	0.0013	0.0235	0.0258
$\sigma_{z,T}$	I-G	0.1	Inf.	0.0174	0.0173	0.0058	0.0164	0.0185
σ_{ξ}	I-G	0.1	Inf.	0.0449	0.0449	0.1867	0.0418	0.0481
σ_{z^I}	I-G	0.1	Inf.	0.2037	0.2049	0.0098	0.1915	0.2144
σ_g	I-G	0.1	Inf.	0.0345	0.0346	0.0020	0.0326	0.0364

Note: "I-G" denotes Inverse-Gamma distribution.

Table A.5. Bayesian Estimation of Structural Parameters : Specification 3

Parameter	Prior			Posterior				
	Distribution	Mean	Std.	Mean	Median	Std.	Percentile 10%	Percentile 90%
γ	Gamma	2	1	2.2855	2.2860	0.0253	2.2812	2.2891
κ	Gamma	4	2	5.6500	5.6499	0.0067	5.6405	5.6604
b_c	Beta	0.5	0.2	0.5324	0.5324	0.0014	0.5320	0.5329
b_l	Beta	0.5	0.2	0.6949	0.6949	0.0021	0.6944	0.6955
χ	Gamma	4	2	2.5873	2.5860	0.0102	2.5826	2.5934
δ_2	Gamma	0.1	0.05	0.0801	0.0801	0.0002	0.800	0.0801
$\phi_1^{x,T}$	Beta	0.6	0.3	0.5510	0.5509	0.0063	0.5504	0.5516
$\phi_2^{x,T}$	Beta	0.2	0.1	0.2990	0.2990	0.0017	0.2988	0.2993
$\phi_1^{a,T}$	Beta	0.6	0.3	0.3462	0.3462	0.0015	0.3456	0.3466
$\phi_2^{a,T}$	Beta	0.2	0.1	0.1491	0.1492	0.0009	0.1486	0.1494
$\phi_1^{z,T}$	Beta	0.6	0.3	0.7377	0.7378	0.0018	0.7360	0.7388
$\phi_2^{z,T}$	Beta	0.2	0.1	0.1306	0.1306	0.0012	0.1305	0.1308
ϕ_1^ξ	Beta	0.6	0.3	0.4461	0.4461	0.0088	0.4446	0.4474
ϕ_2^ξ	Beta	0.2	0.1	0.2181	0.2181	0.0014	0.2179	0.2182
$\phi_1^{z^I}$	Beta	0.6	0.3	0.6521	0.6520	0.0022	0.6516	0.6527
$\phi_2^{z^I}$	Beta	0.2	0.1	0.3431	0.3431	0.0014	0.3428	0.3433
ϕ_1^g	Beta	0.6	0.3	0.6905	0.6905	0.0120	0.6903	0.6906
ϕ_2^g	Beta	0.2	0.1	0.3094	0.3094	0.0011	0.3093	0.3095
ρ^{xg}	Beta	0.7	0.3	0.8524	0.8526	0.0047	0.8511	0.8535
$\sigma_{x,P}$	I-G	0.02	Inf.	0.0082	0.0082	0.0107	0.0077	0.0087
$\sigma_{x,T}$	I-G	0.1	Inf.	0.0387	0.0383	0.0027	0.0372	0.0408
$\sigma_{a,P}$	/	/	/	/	/	/	/	/
$\sigma_{a,T}$	I-G	0.1	Inf.	0.0344	0.0344	0.0017	0.0336	0.0353
$\sigma_{z,P}$	I-G	0.02	Inf.	0.0251	0.0251	0.0014	0.0246	0.0257
$\sigma_{z,T}$	I-G	0.1	Inf.	0.0274	0.0274	0.0012	0.0268	0.0278
σ_ξ	I-G	0.1	Inf.	0.1013	0.1009	0.0051	0.0970	0.1070
σ_{z^I}	I-G	0.1	Inf.	0.1450	0.1457	0.0072	0.1410	0.1474
σ_g	I-G	0.1	Inf.	0.0201	0.0200	0.0209	0.0187	0.0217

Note: "I-G" denotes Inverse-Gamma distribution.

Table A.6. Bayesian Estimation of Structural Parameters : Specification 4

Parameter	Prior			Posterior				
	Distribution	Mean	Std.	Mean	Median	Std.	Percentile 10%	Percentile 90%
γ	Gamma	2	1	1.3097	1.3067	0.0016	1.2861	1.3380
κ	Gamma	4	2	3.3802	3.3712	0.0069	3.3376	3.4396
b_c	Beta	0.5	0.2	0.2196	0.2187	0.0004	0.2115	0.2292
b_l	Beta	0.5	0.2	0.2627	0.2640	0.0005	0.2549	0.2694
χ	Gamma	4	2	4.9082	4.9025	0.0055	4.8614	4.9652
δ_2	Gamma	0.1	0.05	0.0374	0.0374	0.0001	0.0332	0.0409
$\phi_1^{x,T}$	Beta	0.6	0.3	0.3014	0.3029	0.0007	0.2908	0.3099
$\phi_2^{x,T}$	Beta	0.2	0.1	0.2689	0.2667	0.0003	0.2586	0.2834
$\phi_1^{a,T}$	Beta	0.6	0.3	0.6842	0.6839	0.0005	0.6812	0.6874
$\phi_2^{a,T}$	Beta	0.2	0.1	0.2635	0.2635	0.0003	0.2620	0.2652
$\phi_1^{z,T}$	Beta	0.6	0.3	0.4784	0.4782	0.0003	0.4747	0.4825
$\phi_2^{z,T}$	Beta	0.2	0.1	0.2420	0.2417	0.0002	0.2392	0.2450
ϕ_1^ξ	Beta	0.6	0.3	0.8239	0.8239	0.0010	0.8207	0.8267
ϕ_2^ξ	Beta	0.2	0.1	0.1670	0.1668	0.0002	0.1646	0.1702
$\phi_1^{z^I}$	Beta	0.6	0.3	0.5932	0.5931	0.0008	0.5805	0.6062
$\phi_2^{z^I}$	Beta	0.2	0.1	0.2063	0.2033	0.0002	0.1975	0.2191
ϕ_1^g	Beta	0.6	0.3	0.6162	0.6145	0.0006	0.6044	0.6334
ϕ_2^g	Beta	0.2	0.1	0.1502	0.1512	0.0002	0.1455	0.1535
ρ^{xg}	Beta	0.7	0.3	0.1578	0.1621	0.0015	0.1373	0.1731
$\sigma_{x,P}$	/	/	/	/	/	/	/	/
$\sigma_{x,T}$	I-G	0.1	Inf.	0.0341	0.0340	0.0019	0.0322	0.0361
$\sigma_{a,P}$	/	/	/	/	/	/	/	/
$\sigma_{a,T}$	I-G	0.1	Inf.	0.0133	0.0133	0.0003	0.0124	0.0143
$\sigma_{z,P}$	I-G	0.02	Inf.	0.0290	0.0289	0.0008	0.0271	0.0309
$\sigma_{z,T}$	I-G	0.1	Inf.	0.0168	0.0168	0.0016	0.0158	0.0180
σ_ξ	I-G	0.1	Inf.	0.0516	0.0515	0.0170	0.0484	0.0550
σ_{z^I}	I-G	0.1	Inf.	0.1456	0.1455	0.0279	0.1321	0.1592
σ_g	I-G	0.1	Inf.	0.0466	0.0465	0.0012	0.0434	0.0500

Note: "I-G" denotes Inverse-Gamma distribution.

A.5. Supplemental Figures to Conditional Variance Decomposition

Figure A.1. Conditional Variance Decomposition : Baseline

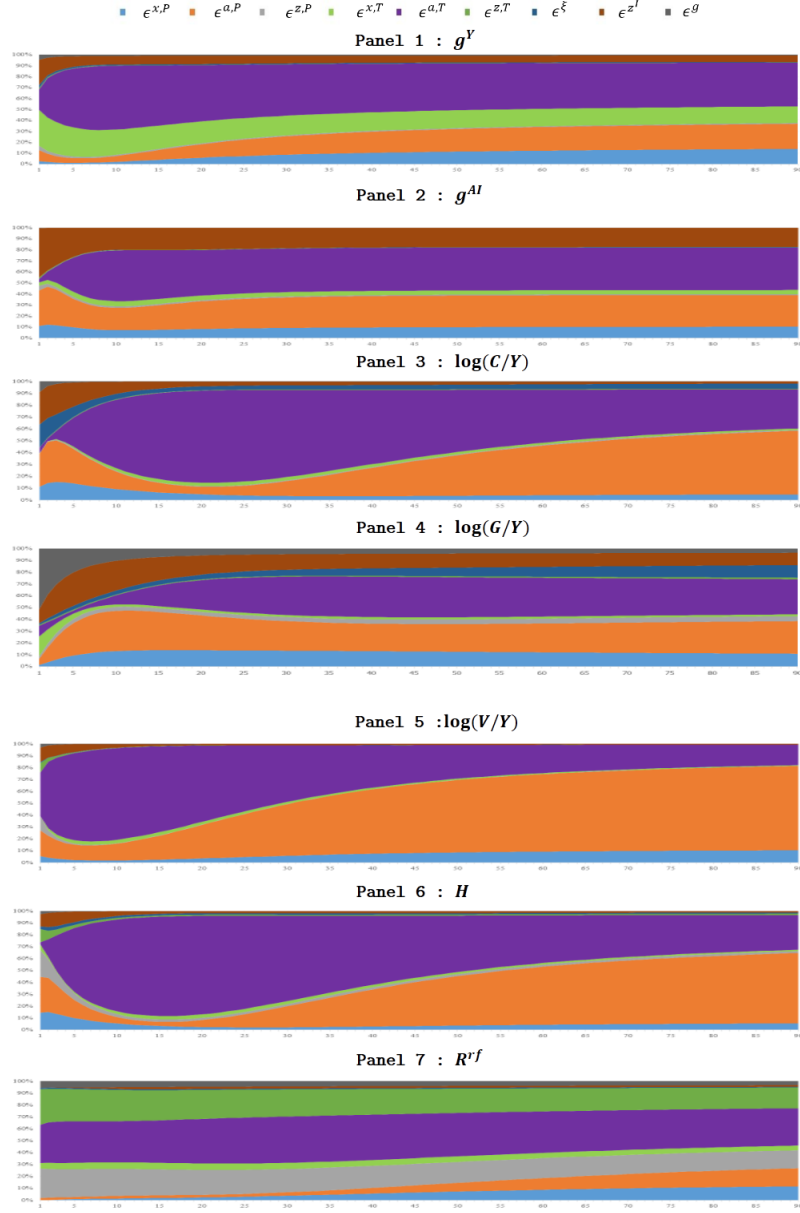


Figure A.2. Conditional Variance Decomposition : Specification 2

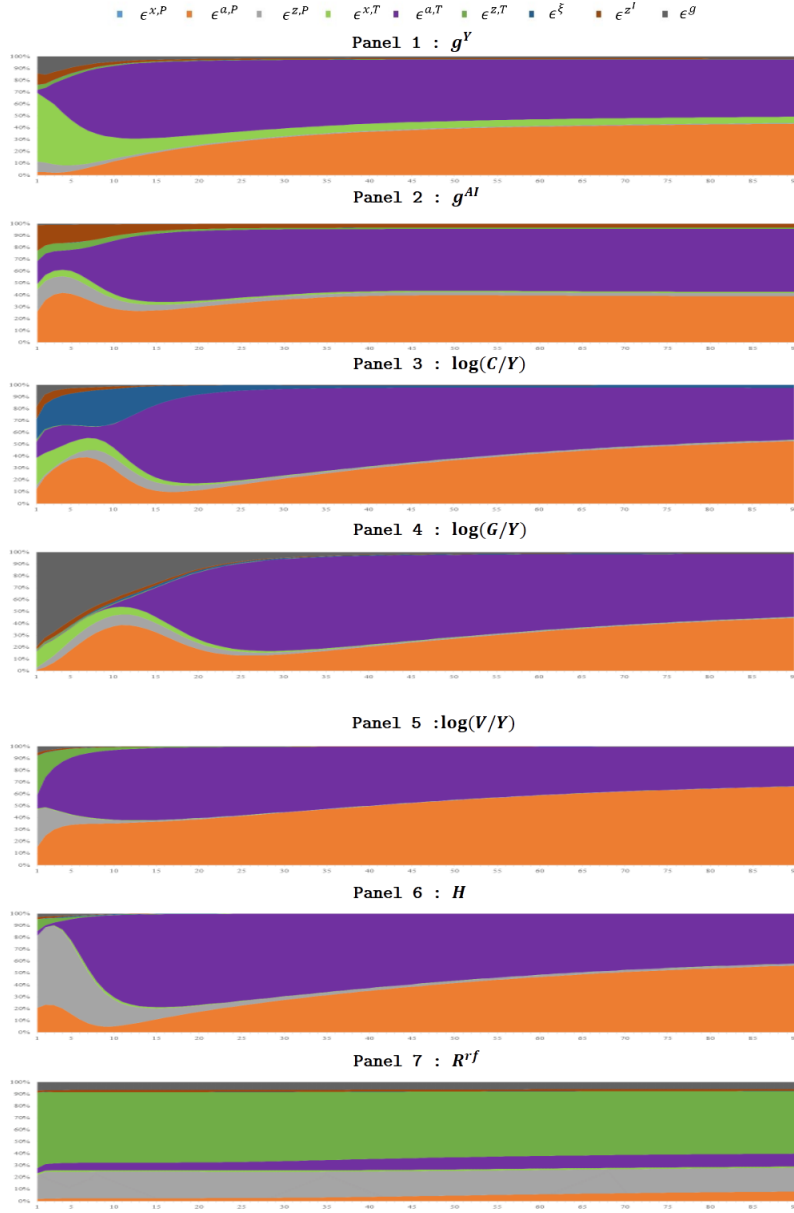


Figure A.3. Conditional Variance Decomposition : Specification 3

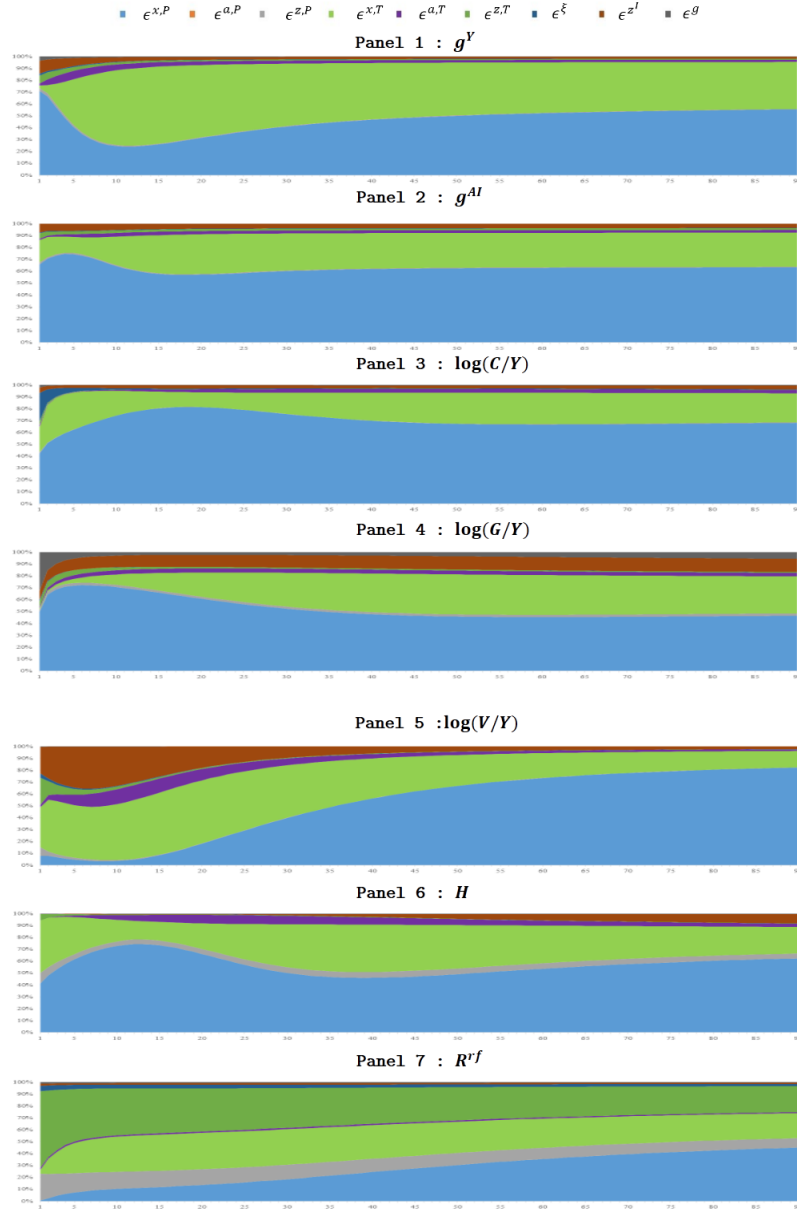
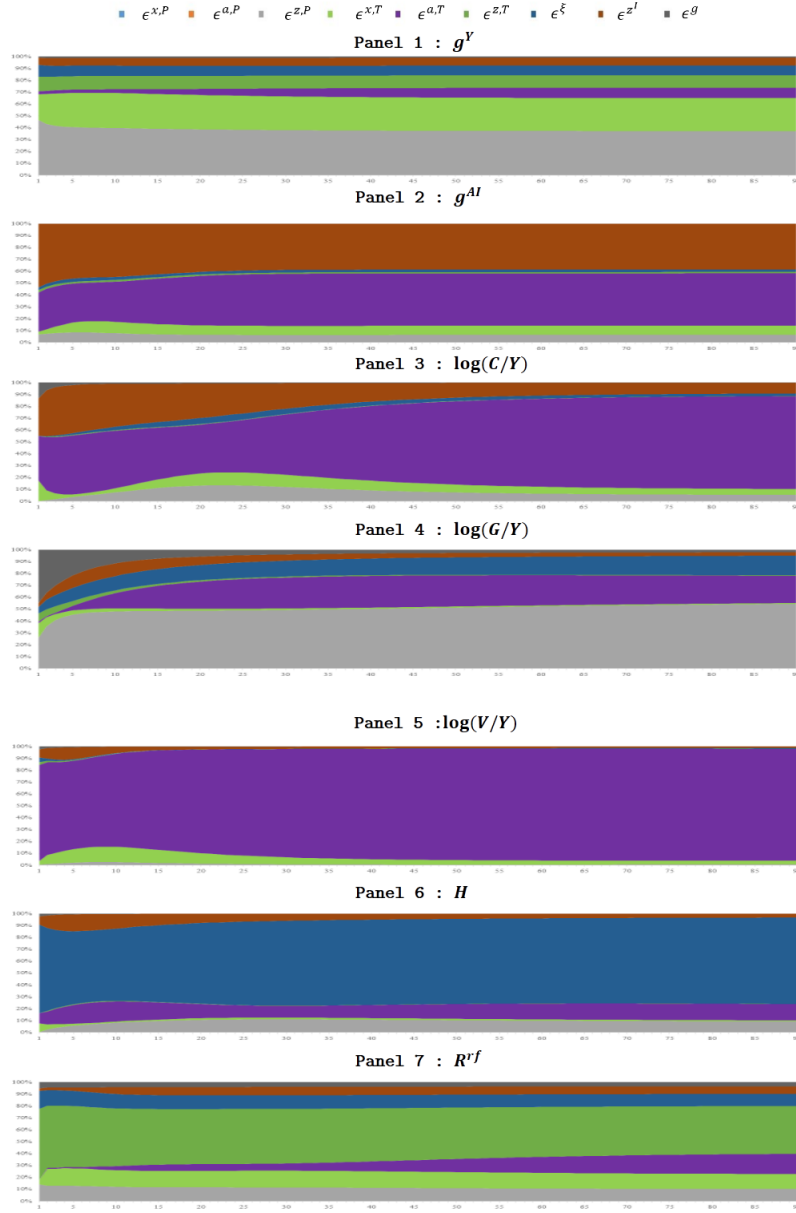


Figure A.4. Conditional Variance Decomposition : Specification 4



A.6. Model Fit and Smoothed Variables

Figure A.5. Actual and Smoothed Variables : Baseline

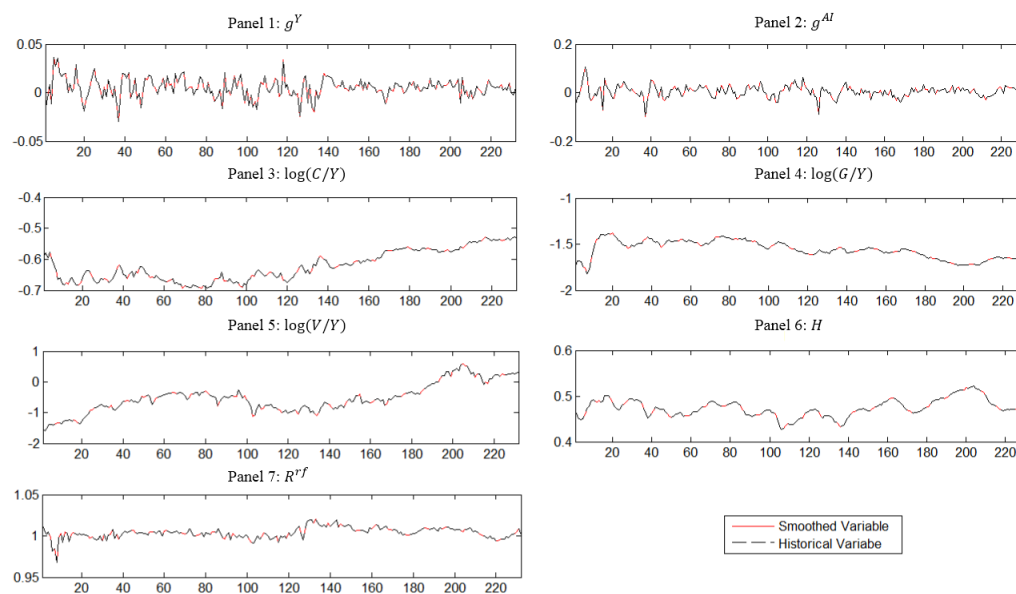


Figure A.6. Smoothed Shocks : Baseline

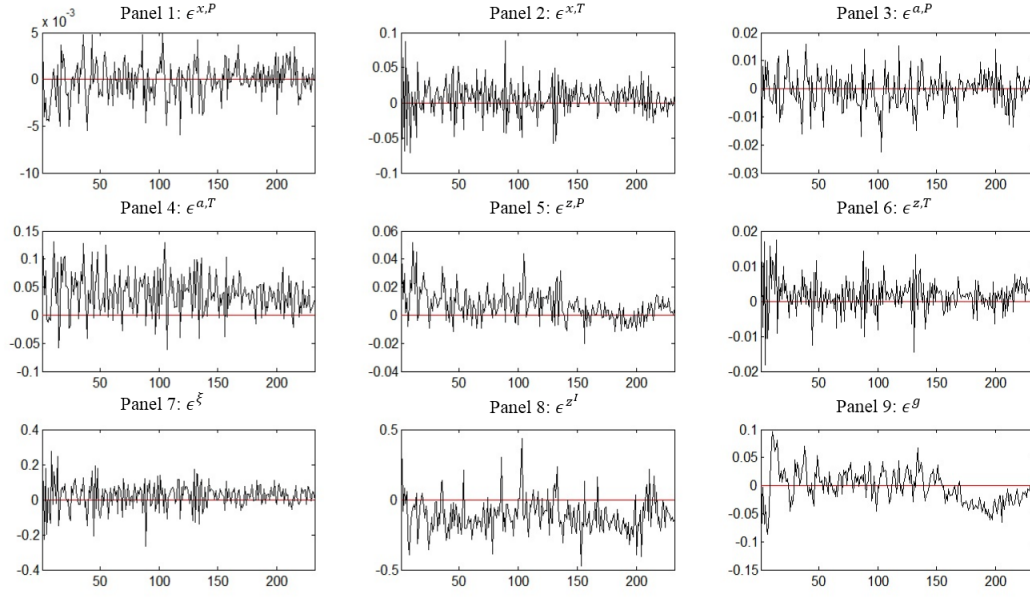


Figure A.7. Actual and Smoothed Variables : Specification 2

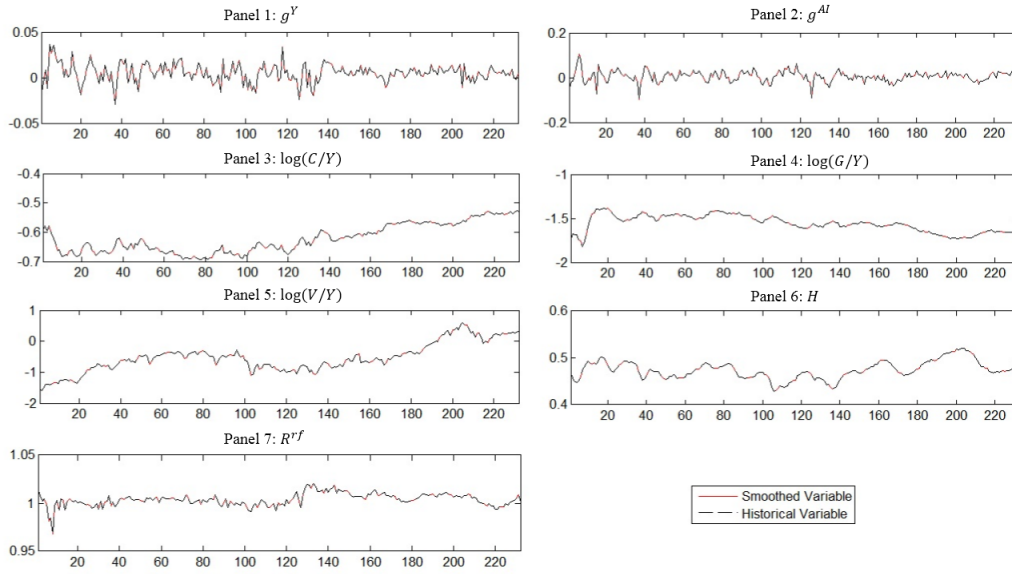


Figure A.8. Smoothed Shocks : Specification 2

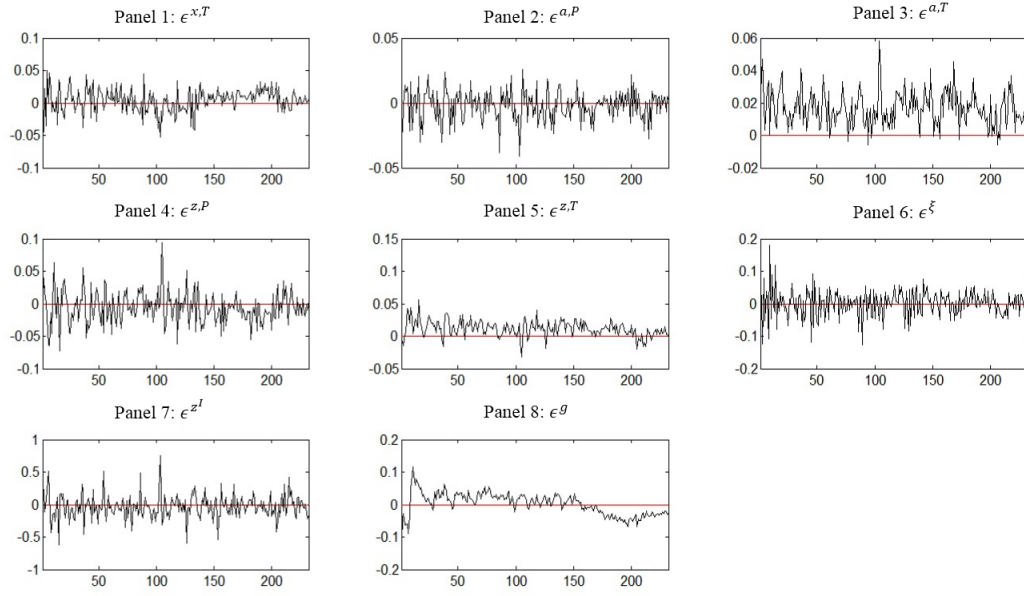


Figure A.9. Actual and Smoothed Variables : Specification 3

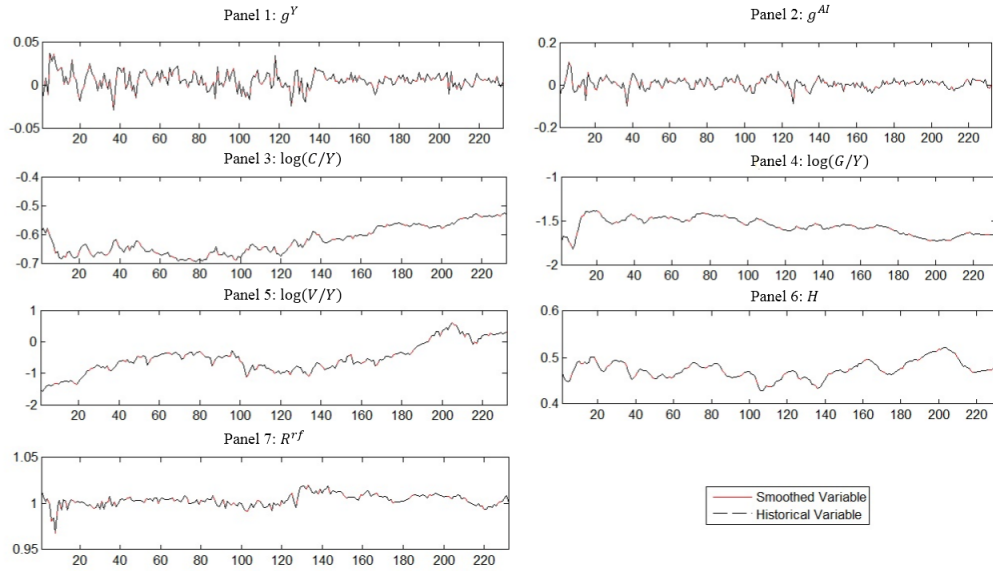


Figure A.10. Smoothed Shocks : Specification 3

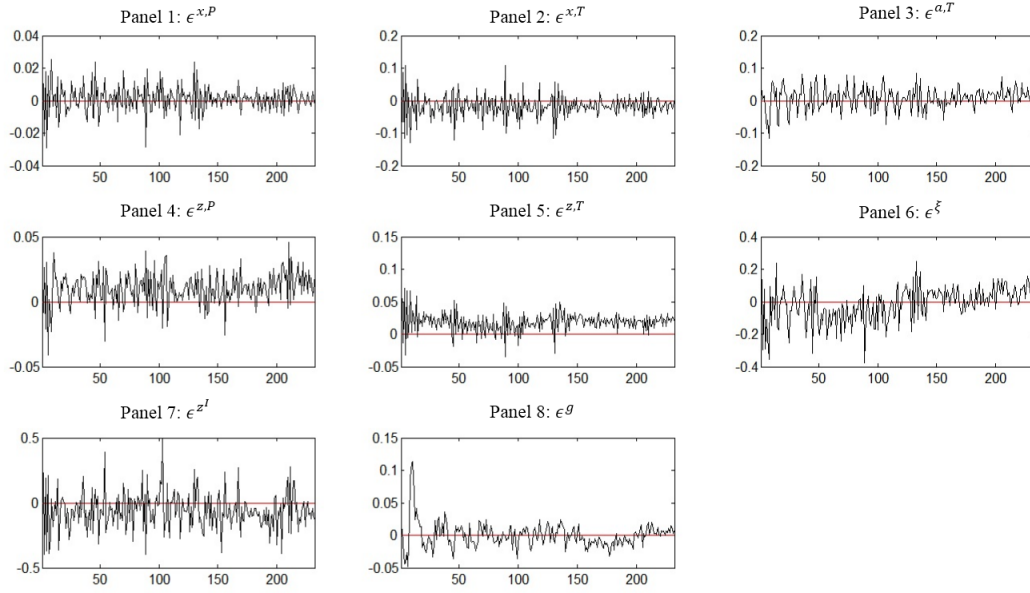


Figure A.11. Actual and Smoothed Variables : Specification 4

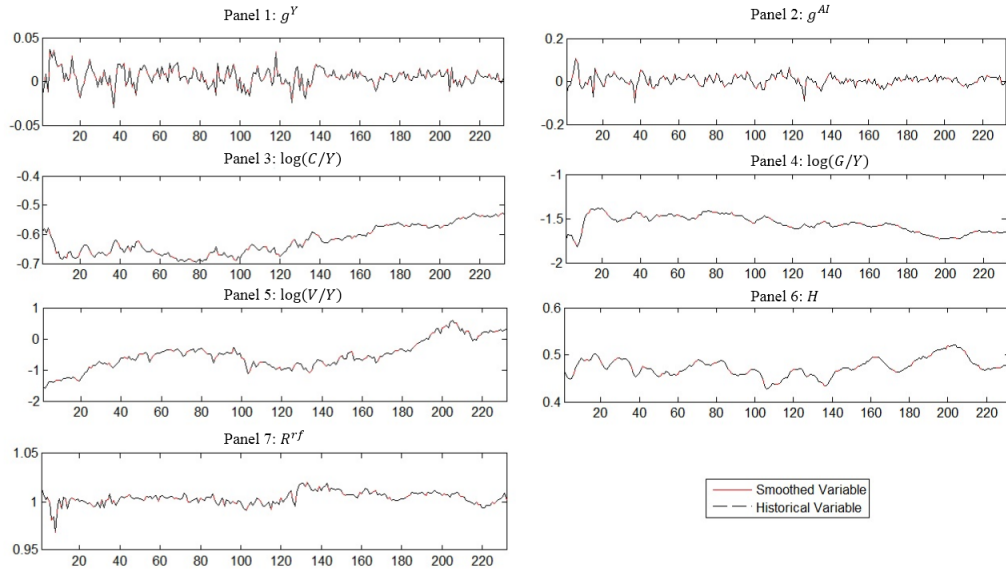
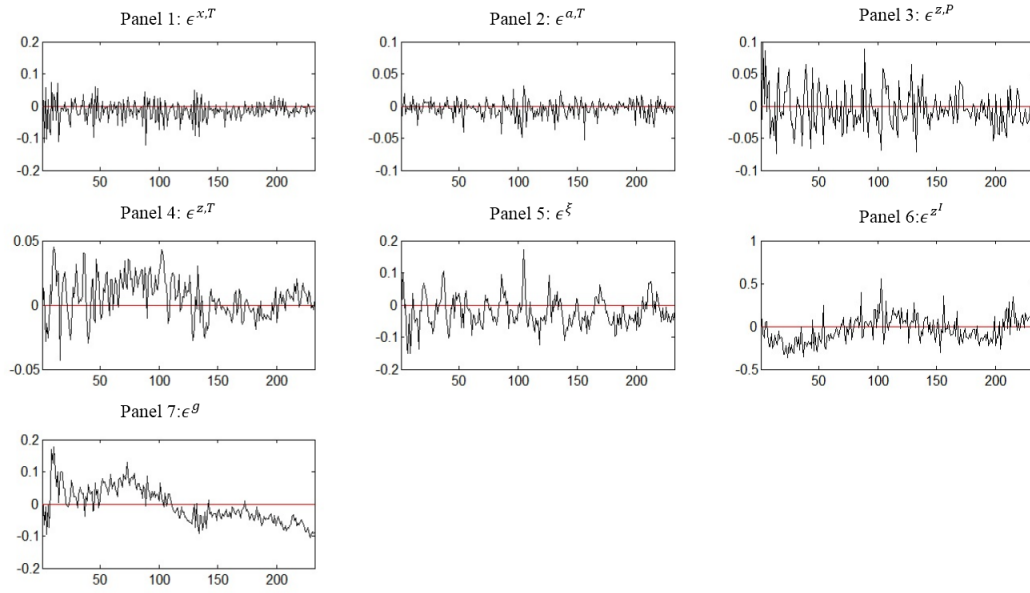


Figure A.12. Smoothed Shocks : Specification 4



A.7. Supplemental Figures to Impulse Responses Analysis

Figure A.13. Dynamic Responses to $\epsilon^{a.P}$

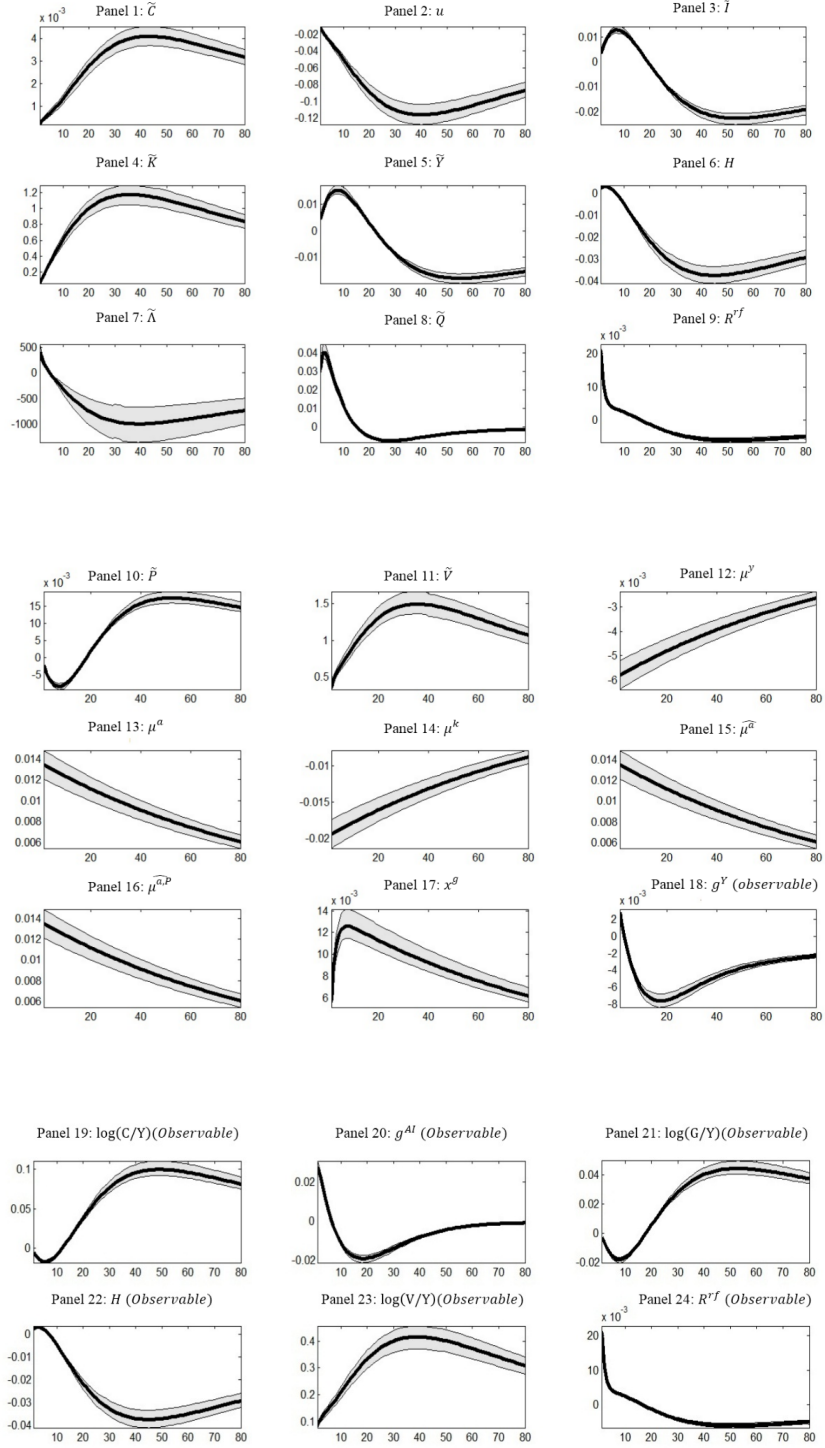


Figure A.14. Dynamic Responses to $\epsilon^{a.T}$

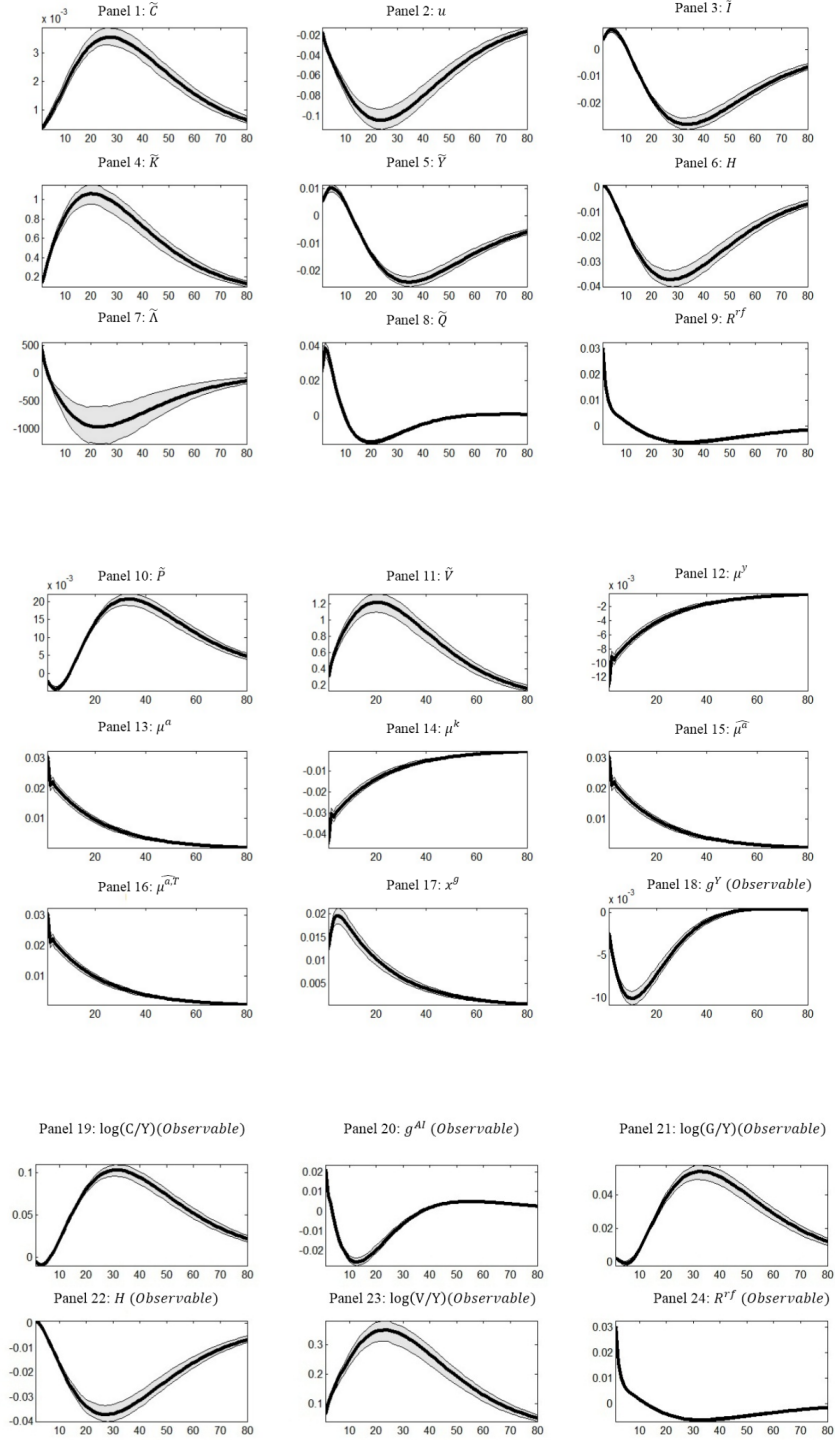


Figure A.15. Dynamic Responses to $\epsilon^{z.P}$

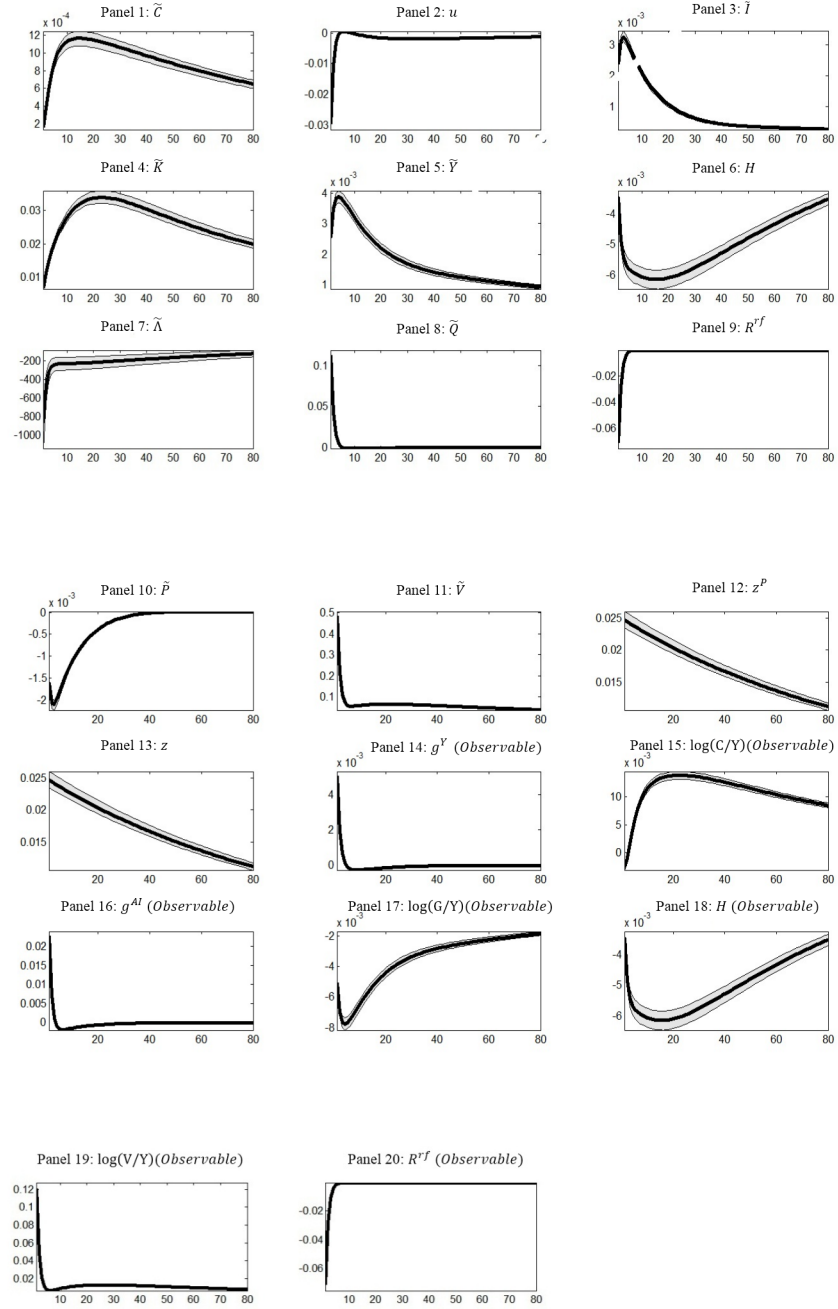


Figure A.16. Dynamic Responses to $\epsilon^{z.T}$

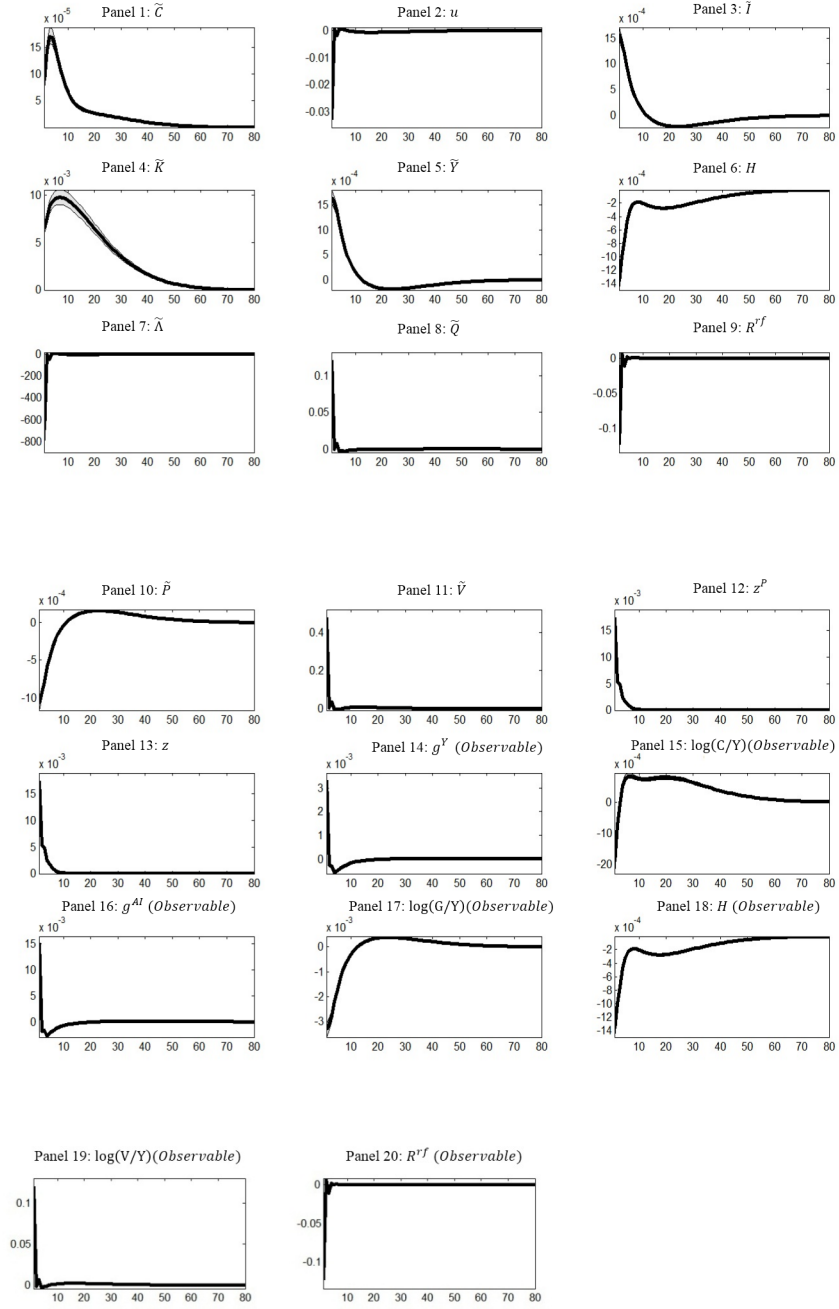


Figure A.17. Dynamic Responses to $\epsilon^{x.T}$

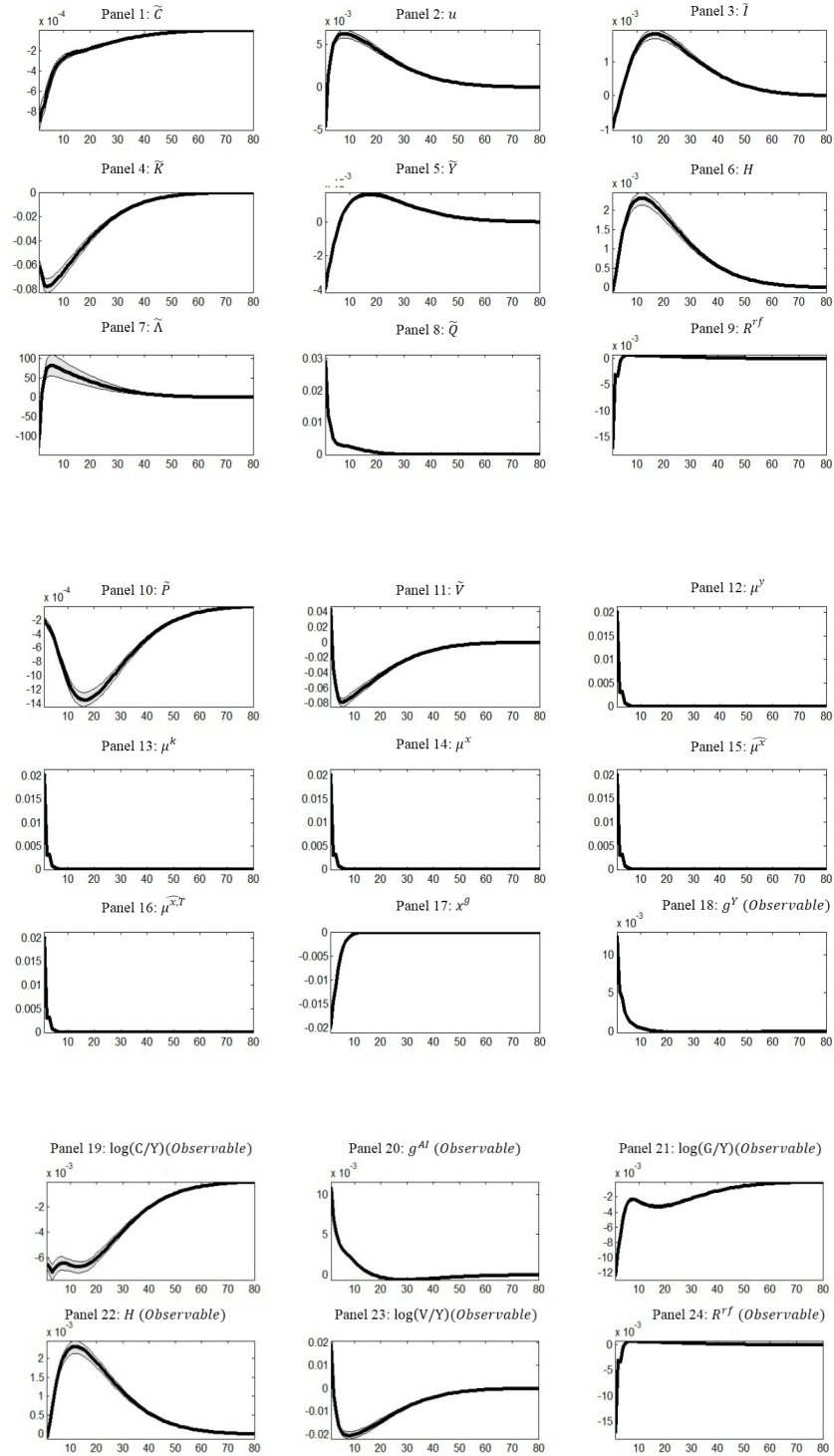


Figure A.18. Dynamic Responses to ϵ^ξ

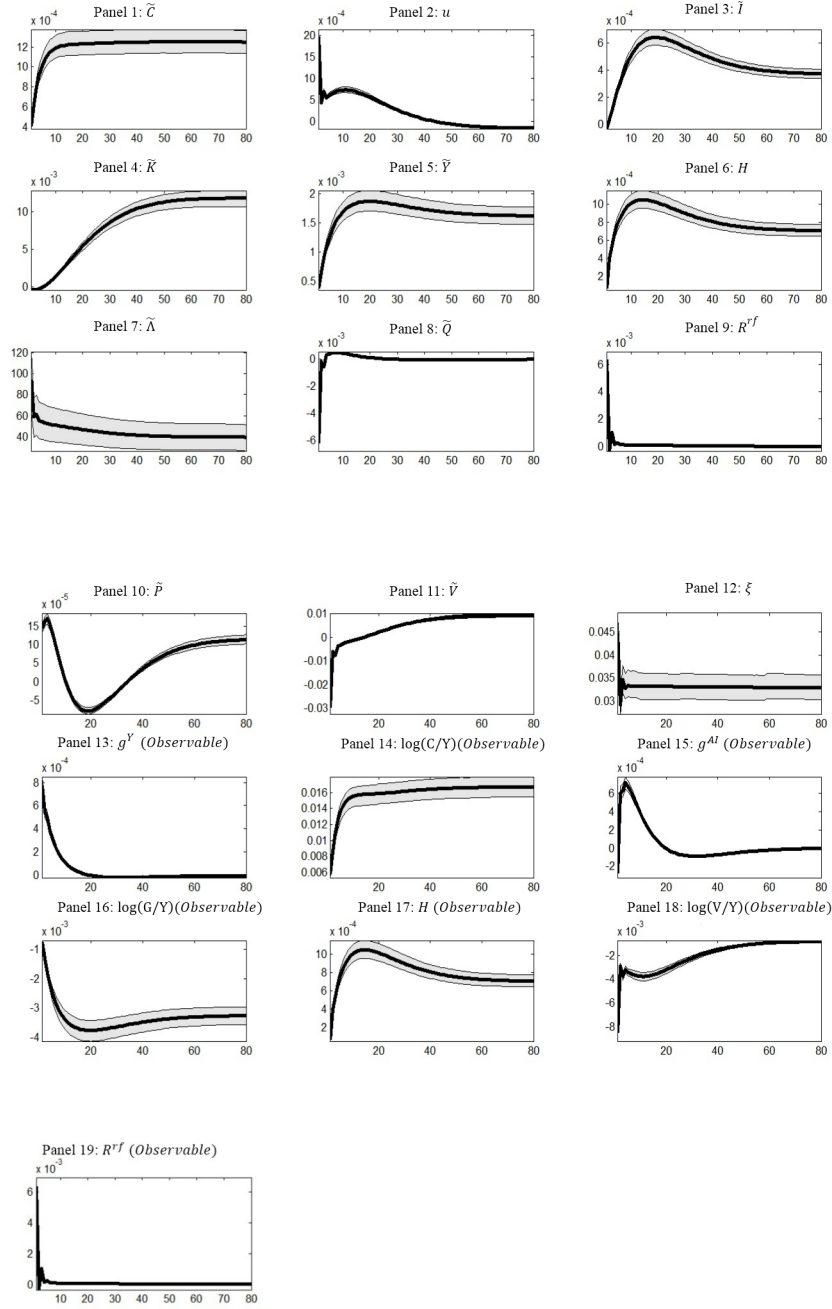


Figure A.19. Dynamic Responses to ϵ^{z^I}

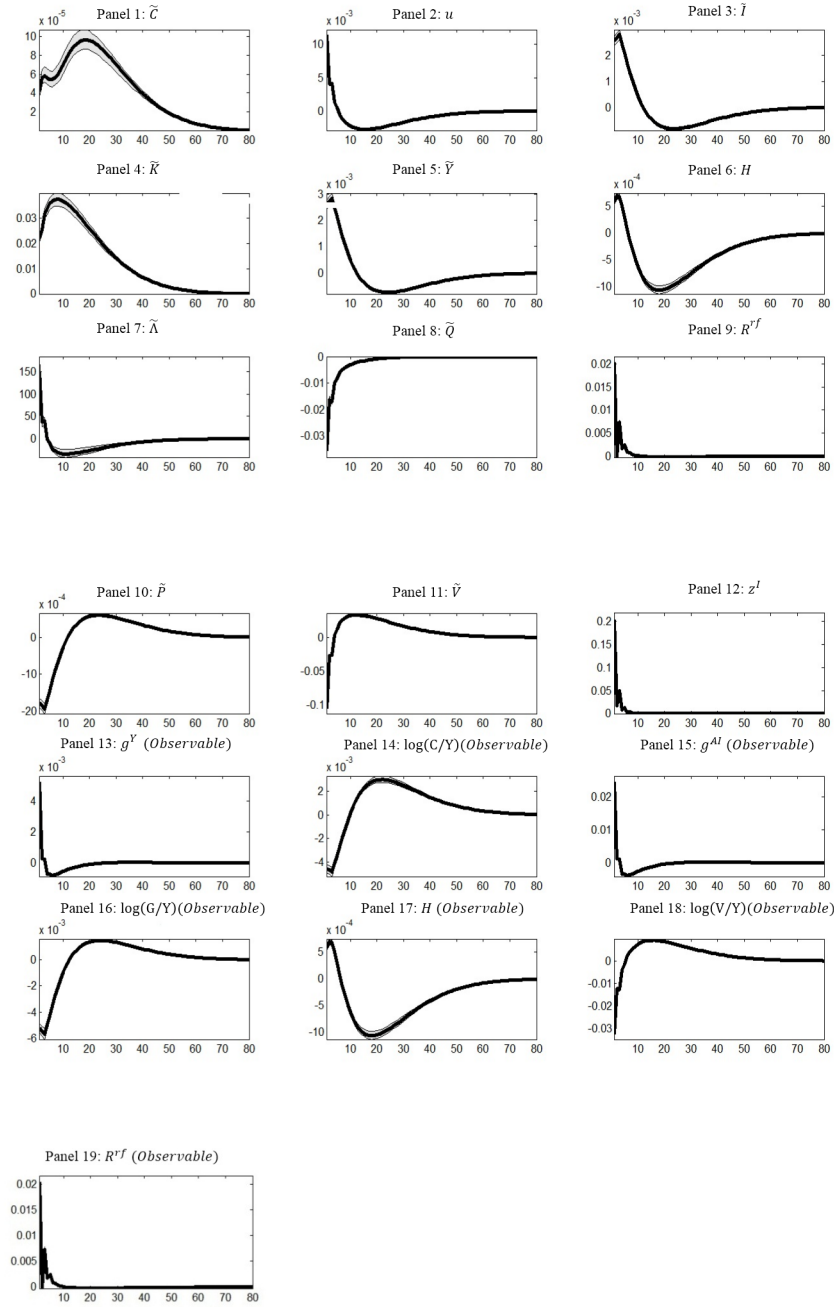
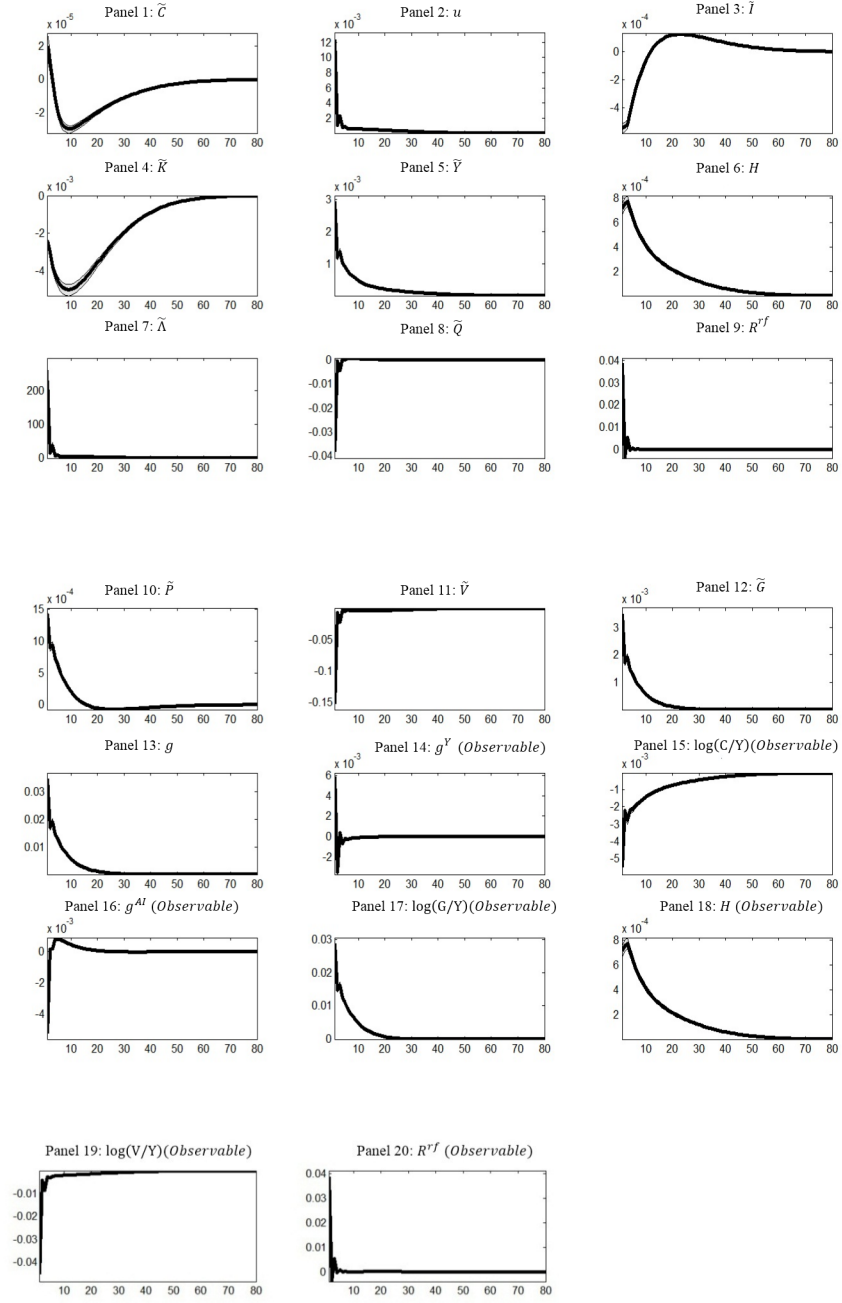


Figure A.20. Dynamic Responses to ϵ^g



APPENDIX B

Appendix of Chapter 2

B.1. Model Solution

Following Albuquerque, Eichenbaum and Rebelo (2013), the logarithm of the return to any dividend-paying asset in our modeled economy satisfies the following Euler condition:

$$E_t [\exp(m_{t+1} + r_{i,t+1})] = 1, \quad (\text{B.1})$$

where the logarithm of the stochastic discount factor m_{t+1} is given by

$$m_{t+1} = \theta \log(\delta) + \theta h_{t+1} - \left(\frac{\theta}{\psi}\right) \Delta c_{t+1} + (\theta - 1) r_{c,t+1}. \quad (\text{B.2})$$

To capture the long-run consumption and valuation risks embedded in the low-frequency movements in consumption growth, dividend growth and time preference dynamics, we specify the exogenous processes as follows:

$$x_{t+1} = \rho x_t + \varphi_e \sigma_t e_{t+1} \quad (\text{B.3})$$

$$h_{t+1} = \rho_\lambda h_t + \sigma_{\lambda,t} \varepsilon_{t+1} \quad (\text{B.4})$$

$$\Delta c_{t+1} = \mu + x_t + \sigma_t \eta_{t+1} \quad (\text{B.5})$$

$$\Delta d_{t+1} = \mu_d + \phi x_t + \varphi_d \sigma_t u_{t+1} \quad (\text{B.6})$$

$$\sigma_{t+1}^2 = \sigma_0^2 + v_1(\sigma_t^2 - \sigma_0^2) + \sigma_w w_{t+1} \quad (\text{B.7})$$

$$\sigma_{\lambda,t+1}^2 = \sigma_\lambda^2 + v_\lambda(\sigma_{\lambda,t}^2 - \sigma_\lambda^2) + \sigma_\pi \pi_{t+1} \quad (\text{B.8})$$

Applying Campbell and Shiller (1989) approximation to the return on aggregate consumption claim $R_{c,t+1}$ and the return on equity asset $R_{d,t+1}$ yields

$$r_{c,t+1} = k_{0,c} + k_{1,c}z_{c,t+1} - z_{c,t} + \Delta c_{t+1} \quad (\text{B.9})$$

$$r_{d,t+1} = k_{0,d} + k_{1,d}z_{d,t+1} - z_{d,t} + \Delta d_{t+1} \quad (\text{B.10})$$

where the constant parameters $k_{0,j}$ and $k_{1,j}$ ($j = c, d$) are defined in equations (2.16) and (2.17).

Since consumption and dividend growth in our modeled economy are purely driven by the unobserved state variables, a complete characterization of the asset returns requires us to pin down the relationship between the state variables and $z_{j,t}$ for $j = c, d$. Our strategy is to solve the price-consumption ratio $z_{c,t}$ first. Conjecture that $z_{c,t}$ is a linear combination of x_t , h_t , σ_t^2 , and $\sigma_{\lambda,t}^2$:

$$z_{c,t} = A_0 + A_1x_t + A_2h_t + A_3\sigma_t^2 + A_4\sigma_{\lambda,t}^2. \quad (\text{B.11})$$

As the return on the consumption claim asset must satisfy the Euler equation (B.1), let $r_{i,t+1} = r_{c,t+1}$ and plug equations (B.2), (B.9) and (B.11) into (B.1). Then we have

$$\begin{aligned} 1 = & E_t \left\{ \exp \left[\theta \log(\delta) + \theta k_{0,c} + \theta k_{1,c}A_0 - \theta k_{1,c}A_0 + \left(\theta - \frac{\theta}{\psi} \right) \mu \right. \right. \\ & + \theta k_{1,c}A_3 (1 - v_1) \sigma^2 + \theta k_{1,c}A_4 (1 - v_{1,\lambda}) \sigma_{\lambda}^2 \\ & + \left(\theta - \frac{\theta}{\psi} - -\theta A_1 + \theta k_{1,c}A_1\rho \right) x_t \\ & + (\theta \rho_{\lambda} + \theta k_{1,c}A_2\rho_{\lambda} - \theta A_2) h_t + \theta A_3 (k_{1,c}v_1 - 1) \sigma_t^2 \\ & + \theta A_4 (k_{1,c}v_{\lambda} - 1) \sigma_{\lambda,t}^2 + \theta k_{1,c}A_1\varphi_e e_{t+1} + \left(\theta - \frac{\theta}{\psi} \right) \sigma_t \eta_{t+1} \\ & \left. \left. + \theta (k_{1,c}A_2 + 1) \sigma_{\lambda,t} \varepsilon_{t+1} + \theta k_{1,c}A_3 \sigma_w w_{t+1} + \theta k_{1,c}A_4 \sigma_{\pi} \pi_{t+1} \right] \right\}. \end{aligned} \quad (\text{B.12})$$

As equation (B.12) must hold for any values of the state variables and the exogenous shocks at any time t , the A 's can be solved by matching-up the undetermined coefficients. It turns

out that the coefficients are given by

$$\begin{aligned}
A_0 &= \left(\frac{1}{1-k_{1,c}} \right) \left[\log(\delta) + k_{0,c} + \left(1 - \frac{1}{\psi} \right) \mu + k_{1,c} A_3 (1 - v_1) \sigma^2 \right. \\
&\quad \left. + k_{1,c} A_4 (1 - v_\lambda) \sigma_\lambda^2 + \left(\frac{\theta}{2} \right) (k_{1,c} A_3)^2 \sigma_w^2 + \left(\frac{\theta}{2} \right) (k_{1,c} A_4)^2 \sigma_\pi^2 \right], \\
A_1 &= \frac{1 - \frac{1}{\psi}}{1 - k_{1,c} \rho}, \\
A_2 &= \frac{\rho_\lambda}{1 - k_{1,c} \rho_\lambda}, \\
A_3 &= \frac{\theta}{2(1 - k_{1,c} v_1)} \left[\left(1 - \frac{1}{\psi} \right)^2 + (k_{1,c} A_1 \varphi_e)^2 \right], \\
A_4 &= \frac{\theta (1 + k_{1,c} A_2)^2}{2(1 - k_{1,c} v_\lambda)}.
\end{aligned}$$

Note that once we obtain the solution to the A 's, the return to the consumption claim asset is solved automatically. Rewrite $r_{c,t+1}$ in terms of the state variables and the innovations:

$$\begin{aligned}
r_{c,t+1} &= [k_{0,c} + k_{1,c} A_0 - A_0 + \mu + k_{1,c} A_3 (1 - v_1) \sigma^2 + k_{1,c} A_4 (1 - v_{1,\lambda}) \sigma_\lambda^2] \\
&\quad + (k_{1,c} A_1 \rho - A_1 + 1) x_t + (k_{1,c} A_2 \rho_\lambda - A_2) h_t + (k_{1,c} A_3 v_1 - A_3) \sigma_t^2 \\
&\quad + (k_{1,c} A_4 v_\lambda - A_4) \sigma_{\lambda,t}^2 + k_{1,c} A_1 \varphi_e e_{t+1} + \sigma_t \eta_{t+1} + k_{1,c} A_2 \sigma_{\lambda,t} \varepsilon_{t+1} \\
&\quad + k_{1,c} A_3 \sigma_w w_{t+1} + k_{1,c} A_4 \sigma_\pi \pi_{t+1}.
\end{aligned} \tag{B.13}$$

Then, we derive the difference between $r_{c,t+1}$ and its conditional expectation:

$$\begin{aligned}
r_{c,t+1} - E_t(r_{c,t+1}) &= k_{1,c} A_1 \varphi_e \sigma_t e_{t+1} + \sigma_t \eta_{t+1} + k_{1,c} A_2 \sigma_{\lambda,t} \varepsilon_{t+1} \\
&\quad + k_{1,c} A_3 \sigma_w w_{t+1} + k_{1,c} A_4 \sigma_\pi \pi_{t+1}.
\end{aligned} \tag{B.14}$$

Therefore, the conditional variance of $r_{c,t+1}$ is given by

$$\begin{aligned} var_t(r_{d,t+1}) &= [(k_{1,c}A_1\varphi_e)^2 + 1] \sigma_t^2 + (k_{1,c}A_2)^2 \sigma_{\lambda,t}^2 + (k_{1,c}A_3)^2 \sigma_w^2 \\ &\quad + (k_{1,c}A_4)^2 \sigma_\pi^2. \end{aligned} \tag{B.15}$$

Similarly, we derive the difference between m_{t+1} and its conditional expectation:

$$\begin{aligned} m_{t+1} - E_t(m_{t+1}) &= (\theta - 1) k_1 A_1 \varphi_e \sigma_t e_{t+1} + \left(\theta - \frac{\theta}{\psi} - 1 \right) \sigma_t \eta_{t+1} \\ &\quad [\theta + (\theta - 1) k_1 A_2] \sigma_{\lambda,t} \varepsilon_{t+1} + (\theta - 1) k_1 A_3 \sigma_w w_{t+1} \\ &\quad + (\theta - 1) k_1 A_4 \sigma_\pi \pi_{t+1} \end{aligned} \tag{B.16}$$

$$\begin{aligned} &= -\beta_{m,e} \sigma_t e_{t+1} - \beta_{m,\eta} \sigma_t \eta_{t+1} - \beta_{m,\varepsilon} \sigma_{\lambda,t} \varepsilon_{t+1} \\ &\quad - \beta_{m,w} \sigma_w w_{t+1} - \beta_{m,\pi} \sigma_\pi \pi_{t+1}. \end{aligned}$$

Therefore, it is straightforward to calculate the conditional variance of m_{t+1} , which is given by

$$var_t(m_{t+1}) = (\beta_{m,e}^2 + \beta_{m,\eta}^2) \sigma_t^2 + \beta_{m,\varepsilon}^2 \sigma_{\lambda,t}^2 + \beta_{m,w}^2 \sigma_w^2 + \beta_{m,\pi}^2 \sigma_\pi^2. \tag{B.17}$$

Using equations (B.14), (B.15) and (B.16), we are able to derive the risk premium for the consumption claim asset. That is,

$$\begin{aligned}
E_t(r_{c,t+1} - r_{f,t}) &= -cov_t(m_{t+1} - E_t(m_{t+1}), r_{c,t+1} - E_t(r_{c,t+1})) \\
&\quad - 0.5var_t(r_{c,t+1}) \\
&= -\left\{ \left[\left(\theta - \frac{1}{2} \right) (k_{1,c} A_1 \varphi_e)^2 + \left(\theta - \frac{\theta}{\psi} \right) \right] \sigma_t^2 \right. \\
&\quad \left. + \left[\left(\theta - \frac{1}{2} \right) (k_{1,c} A_2)^2 + \theta k_{1,c} A_2 \right] \sigma_{\lambda,t}^2 \right. \\
&\quad \left. + \left(\theta - \frac{1}{2} \right) (k_{1,c} A_3)^2 \sigma_w^2 + \left(\theta - \frac{1}{2} \right) (k_{1,c} A_4)^2 \sigma_\pi^2 \right\}.
\end{aligned} \tag{B.18}$$

After solving for the variables related to the consumption claim asset, we move onto the solution to the equity assets. Conjecture that $z_{d,t}$ takes the following functional form:

$$z_{d,t} = A_{0,d} + A_{1,d}x_t + A_{2,d}h_t + A_{3,d}\sigma_t^2 + A_{4,d}\sigma_{\lambda,t}^2. \tag{B.19}$$

Let $r_{i,t+1} = r_{d,t+1}$ and rewrite the Euler equation as

$$E_t \left\{ \exp \left[\theta \log(\delta) + \theta h_{t+1} - \frac{\theta}{\psi} \Delta c_{t+1} + \theta r_{c,t+1} + (r_{d,t+1} - r_{c,t+1}) \right] \right\} = 1. \tag{B.20}$$

It can be show that

$$\begin{aligned}
r_{d,t+1} - r_{c,t+1} = & \quad [(k_{0,d} - k_0) + (k_{1,d}A_{0,d} - k_{1,c}A_0) - (A_{0,d} - A_0) + (\mu_d - \mu) \\
& + (k_{1,d}A_{3,d} - k_{1,c}A_3) (1 - v_1) \sigma^2 + (k_{1,d}A_{4,d} - k_{1,c}A_4) (1 - v_\lambda) \sigma_\lambda^2] \\
& + [(A_1 - A_{1,d}) + (k_{1,d}A_{1,d} - k_{1,c}A_1) \rho + (\phi - 1)] x_t \\
& + [(A_2 - A_{2,d}) + (k_{1,d}A_{2,d} - k_{1,c}A_2) \rho_\lambda] h_t \\
& + [(A_3 - A_{3,d}) + (k_{1,d}A_{3,d} - k_{1,c}A_3) v_1] \sigma_t^2 \\
& + [(A_4 - A_{4,d}) + (k_{1,d}A_{4,d} - k_{1,c}A_4) v_\lambda] \sigma_{\lambda,t}^2 \\
& + (k_{1,d}A_{1,d} - k_{1,c}A_1) \varphi_e \sigma_t e_{t+1} + (k_{1,d}A_{2,d} - k_{1,c}A_2) \sigma_{\lambda,t} \varepsilon_{t+1} \\
& + (k_{1,d}A_{3,d} - k_{1,c}A_3) \sigma_w w_{t+1} + (k_{1,d}A_{4,d} - k_{1,c}A_4) \sigma_\pi \pi_{t+1} \\
& + \varphi_d \sigma_t u_{t+1} - \sigma_t \eta_{t+1}.
\end{aligned} \tag{B.21}$$

Matching-up the undetermined coefficients we obtain the solution to the A_d 's:

$$\begin{aligned}
A_{0,d} = & \quad \left(\frac{1}{1-k_{1,d}} \right) \left\{ \log(\delta) + k_{0,d} + \mu_d - \frac{\mu}{\psi} + k_{1,d}A_{3,d} (1 - v_1) \sigma^2 \right. \\
& + k_{1,d}A_{4,d} (1 - v_\lambda) \sigma_\lambda^2 + \left[\left(\frac{\theta}{2} \right) (k_{1,d}A_{3,d})^2 + \left(\frac{1}{2} \right) (k_{1,d}A_{3,d} - k_{1,c}A_3)^2 \right] \sigma_w^2 \\
& \left. + \left[\left(\frac{\theta}{2} \right) (k_{1,d}A_{4,d})^2 + \left(\frac{1}{2} \right) (k_{1,d}A_{4,d} - k_{1,c}A_4)^2 \right] \sigma_\pi^2 \right\}, \\
A_{1,d} = & \quad \frac{\phi - \frac{1}{\psi}}{1 - k_{1,d}\rho}, \\
A_{2,d} = & \quad \frac{\rho_\lambda}{1 - k_{1,d}\rho_\lambda}, \\
A_{3,d} = & \quad \frac{1}{2(1 - k_{1,d}v_1)} \left\{ \theta \left(1 - \frac{1}{\psi} \right)^2 + (\varphi_d)^2 + [(k_{1,d}A_{1,d} - k_{1,c}A_1)^2 + (k_{1,c}A_1)^2] (\varphi_e)^2 \right\}, \\
A_{4,d} = & \quad \frac{1}{2(1 - k_{1,d}v_\lambda)} [\theta (1 + k_{1,c}A_2)^2 + (k_{1,d}A_{2,d} - k_{1,c}A_2)^2].
\end{aligned}$$

Then, the solution to $r_{d,t+1}$ can be derived by simply plugging $z_{d,t+1}, z_{d,t}$ and Δd_{t+1} into equation (B.10). Here we derive the difference between $r_{d,t+1}$ and its conditional expectation:

$$\begin{aligned} r_{d,t+1} - E_t(r_{d,t+1}) &= k_{1,d}A_{1,d}\varphi_e\sigma_t e_{t+1} + k_{1,d}A_{2,d}\sigma_{\lambda,t}\varepsilon_{t+1} + k_{1,d}A_{3,d}\sigma_w w_{t+1} \\ &\quad + k_{1,d}A_{4,d}\sigma_\pi\pi_{t+1} + \varphi_d\sigma_t u_{t+1} \end{aligned} \tag{B.22}$$

$$\begin{aligned} &= \beta_{d,e}\sigma_t e_{t+1} + \beta_{d,\varepsilon}\sigma_{\lambda,t}\varepsilon_{t+1} + \beta_{d,w}\sigma_w w_{t+1} \\ &\quad + \beta_{d,\pi}\sigma_\pi\pi_{t+1} + \varphi_d\sigma_t u_{t+1}. \end{aligned}$$

Therefore, the conditional variance of $r_{d,t+1}$ is

$$var_t(r_{d,t+1}) = (\beta_{d,e}^2 + \varphi_d^2)\sigma_t^2 + \beta_{d,\varepsilon}^2\sigma_{\lambda,t}^2 + \beta_{d,w}^2\sigma_w^2 + \beta_{d,\pi}^2\sigma_\pi^2. \tag{B.23}$$

Similarly, using equations (B.16), (B.22) and (B.23) we obtain the equity premium, which is given by

$$\begin{aligned} E_t(r_{d,t+1} - r_{f,t}) &= -cov_t(m_{t+1} - E_t(m_{t+1}), r_{d,t+1} - E_t(r_{d,t+1})) \\ &\quad - 0.5var_t(r_{d,t+1}) \end{aligned} \tag{B.24}$$

$$\begin{aligned} &= \beta_{d,e}\beta_{m,e}\sigma_t^2 + \beta_{d,\varepsilon}\beta_{m,\varepsilon}\sigma_{\lambda,t}^2 + \beta_{d,w}\beta_{m,w}\sigma_w^2 \\ &\quad + \beta_{d,\pi}\beta_{m,\pi}\sigma_\pi^2 - 0.5var_t(r_{d,t+1}). \end{aligned}$$

Finally, to close the model requires solving for the risk-free rate $r_{f,t}$. Guess and verify that $r_{f,t}$ is a linear combination of the state variables:

$$r_{f,t} = A_{0,rf} + A_{1,rf}x_t + A_{2,rf}h_t + A_{3,rf}\sigma_t^2 + A_{4,rf}\sigma_{\lambda,t}^2. \tag{B.25}$$

Analogous to the way that we solve for $z_{d,t}$, rewrite the Euler equation as

$$E_t \left\{ \exp \left[\theta \log(\delta) + \theta h_{t+1} - \frac{\theta}{\psi} \Delta c_{t+1} + \theta r_{c,t+1} + (r_{f,t} - r_{c,t+1}) \right] \right\} = 1. \quad (\text{B.26})$$

Then substitute $r_{f,t}$ using equation (B.25) and match-up the undetermined coefficients. The solution to the A_{rf} 's is

$$\begin{aligned} A_0 = & (k_{0,c} + k_{1,c}A_0 - A_0 + \mu) + k_{1,c}A_3(1 - v_1)\sigma^2 \\ & + k_{1,c}A_4(1 - v_\lambda)\sigma_\lambda^2 - \left(\frac{1}{2}\right)(k_{1,c}A_3)^2\sigma_w^2 - \left(\frac{1}{2}\right)(k_{1,c}A_4)^2\sigma_\pi^2, \end{aligned}$$

$$A_{1,rf} = \frac{1}{\psi},$$

$$A_{2,rf} = -\rho_\lambda,$$

$$A_{3,rf} = -\frac{1}{2} \left[\theta \left(1 - \frac{1}{\psi} \right)^2 + (\theta + 1)(k_{1,c}A_1\varphi_e)^2 + 1 \right],$$

$$A_{4,rf} = -\frac{1}{2} [\theta(1 + k_{1,c}A_2)^2 + (k_{1,c}A_2)^2].$$

In this case, we derive the unconditional variance of the risk-free rate, which is simply

$$\text{var}(r_{f,t}) = A_{1,rf}^2 \text{var}(x_t) + A_{2,rf}^2 \text{var}(h_t) + A_{3,rf}^2 \text{var}(\sigma_t) + A_{4,rf}^2 \text{var}(\sigma_{\lambda,t}^2). \quad (\text{B.27})$$

B.2. Data Construction

B.2.1. Quarterly Data for Parameter Estimates

The quarterly data on consumption growth, dividend growth, price-dividend ratio, market return and risk-free rate is constructed using the following series:

(1) Nominal Gross Domestic Product (Quarterly), downloaded from BEA (www.bea.gov) National Income and Product Accounts Table 1.1.5 (Quarterly), line 1, billions of dollars seasonally adjusted at annual rate;

(2) Real Gross Domestic Product (Quarterly), downloaded from BEA (www.bea.gov), National Income and Product Accounts Table 1.1.6 (Quarterly), line 1, billions of chained 2009 dollars seasonally adjusted at annual rate;

(3) Nominal Personal Consumption on Nondurable Goods (Quarterly), downloaded from BEA (www.bea.gov), National Income and Product Accounts Table 1.1.5 (Quarterly), line 5, billions of dollars seasonally adjusted at annual rate;

(4) Nominal Personal Consumption on Services (Quarterly), downloaded from BEA (www.bea.gov), National Income and Product Accounts Table 1.1.5 (Quarterly), line 6, billions of dollars seasonally adjusted at annual rate;

(5) Civilian Non-institutional Population over 16 (Quarterly), BLS label LNU00000000Q, downloaded from BLS (www.bls.gov);

(6) 3-Month Treasury Bill Secondary Market Rate (Quarterly), downloaded from FRED (research.stlouisfed.org), not seasonally adjusted;

(7) Market Return Including Dividend (Quarterly), CRSP data with all the stocks traded on NYSE, AMEX and NASDAQ included, downloaded from WRDS, thousands of dollars;

(8) Market Return Excluding Dividend (Quarterly), CRSP data with all the stocks traded on NYSE, AMEX and NASDAQ included, downloaded from WRDS, thousands of dollars.

We first define the GDP deflator, $GDPdef_t$, which is given by

$$GDPdef_t = (1)/(2) .$$

The real per capita consumption C_t is therefore defined as

$$C_t = [(3) + (4)] / [GDPdef_t \cdot (5)] .$$

Thus, the real per capita consumption growth Δc_t can be obtained using the following formula:

$$\Delta c_t = \log (C_t / C_{t-1}) .$$

The construction of the real risk-free rate series takes two steps. First, given that the 3-month Treasury Bill rate is measured as annualized return in percentage points, the nominal risk-free rate at quarterly frequency, $tbrate_t^Q$, is defined as

$$tbrate_t^Q = (6) / 400 .$$

Then, we assume that agents perfectly foresee future inflation, and thus calculate the real risk-free $r_{f,t}$ as

$$r_{f,t} = tbrate_t^Q - \log (GDPdef_{t+1} / GDPdef_t) ,$$

where $\log (GDPdef_{t+1} / GDPdef_t)$ measures the expected inflation at period $t + 1$.

For financial variables, we assume that the nominal price of the equity asset at the initial period P_0 is equal to 1. Letting $rmexc = (8)$, we construct the price series using the following formula:

$$P_t = P_{t-1} (1 + rmexc_t) .$$

Using the GDP deflator, we calculate the real price of the equity asset P_t^{real} as

$$P_t^{real} = P_t / GDPdef_t .$$

Then, we construct the series of the real dividend payment. Given the series of P_t , we let $rminc = (7)$ and define the nominal dividend payment at the quarterly frequency, $dividend_t$,

as

$$dividend_t = P_{t-1} (rminc_t - rmexc_t) .$$

Consequently, the real dividend payment $dividend_t^{real}$ is given by

$$dividend_t^{real} = dividend_t / GDPdef_t .$$

Considering that the series of real dividend payment exhibits strong seasonality, real dividend growth and price-dividend ratio would be poorly measured if we use $dividend_t^{real}$ without any adjustments. Following the literature, we take the summation of the real dividend payment over the current and the past three quarters:

$$dividend_t^{adj} = \sum_{i=0}^3 dividend_{t-i}^{real} .$$

Then, the real dividend growth Δd_t is given by

$$\Delta d_t = \log \left(dividend_t^{adj} / dividend_{t-1}^{adj} \right) ,$$

and the logarithm of the price-dividend ratio $z_{d,t}$ is defined as

$$z_{d,t} = \log \left(P_t^{real} / dividend_t^{adj} \right) + \log(4) .$$

In addition, the real market return $r_{m,t}$ takes the following form:

$$r_{m,t} = \log \left[(P_t^{real} + dividend_t^{real}) / P_{t-1}^{real} \right] .$$

Finally, we calculate the realized equity premium (instead of the expected equity premium), which is given by

$$ep_t = r_{m,t} - r_{f,t} .$$

In the robustness test, we use the CPI deflator, rather than the GDP deflator, to construct the real variables. The data management procedure is similar to the one described above.

B.2.2. Monthly Data for Latent Variable Smoothing

The monthly data on dividend growth, price-dividend ratio, market return and risk-free rate is constructed using the following series:

(1) Consumer Price Index (Monthly), downloaded from FRED (research.stlouisfed.org), seasonally adjusted;

(2) Personal Consumption Expenditure, downloaded from FRED (research.stlouisfed.org), seasonally adjusted annual rate;

(3) 3-Month Treasury Bill Secondary Market Rate (Monthly), downloaded from FRED (research.stlouisfed.org), not seasonally adjusted;

(4) Market Return Including Dividend (Monthly), CRSP data with all the stocks traded on NYSE, AMEX and NASDAQ included, downloaded from WRDS, thousands of dollars;

(5) Market Return Excluding Dividend (Monthly), CRSP data with all the stocks traded on NYSE, AMEX and NASDAQ included, downloaded from WRDS, thousands of dollars.

Different from the method described in Appendix B.2.1, we deflate the nominal variables at the monthly frequency using the Consumer Price Index. First, we know that the 3-month Treasury Bill rate at the monthly frequency is given by

$$tbrate_t^M = (2) / 1200 .$$

We let $CPI^M = (1)$ and assume that the agents perfectly foresee future inflation. Hence, the real risk-free rate series can be constructed using the following formula:

$$r_{f,t}^M = tbrate_t^M - \log (CPI_{t+1}^M / CPI_t^M) ,$$

where $\log (CPI_{t+1}^M / CPI_t^M)$ measures the expected inflation at period $t + 1$.

Second, let $C^M = (2)$. Then, the series of real consumption growth is simply given by

$$\Delta c_t^M = \log [(C_t^M / CPI_t^M) / (C_{t-1}^M / CPI_{t-1}^M)] .$$

The construction of the data series on the financial variables at the monthly frequency is analogous to the one described in Appendix B.2.1. Letting $P_0^M = 1$ and $rmexc^M = (5)$, we obtain the price series using the following formula:

$$P_t^M = P_{t-1}^M (1 + rmexc_t^M) .$$

Consequently, the real price of the equity asset is given by

$$P_t^{M,real} = P_t^M / CPI_t^M .$$

Letting $rminc^M = (4)$, the series of the real dividend payment is recovered from the following formula:

$$dividend_t^{M,real} = P_{t-1}^M (rminc_t^M - rmexc_t^M) / CPI_t^M .$$

To remove the seasonality in the dividend series, we take the summation of the real dividend payment over the current and the past eleven months:

$$dividend_t^{M,adj} = \sum_{i=0}^{11} dividend_{t-i}^{M,real} .$$

Finally, we obtain the dividend growth, the price-dividend ratio and the real market return series, which are given by

$$\Delta d_t^M = \log \left(dividend_t^{M,adj} / dividend_{t-1}^{M,adj} \right) ,$$

$$z_{d,t}^M = \log \left(P_t^{M,real} / dividend_t^{M,adj} \right) + \log (12) ,$$

$$r_{m,t}^M = \log \left[\left(P_t^{M,real} + dividend_t^{M,real} \right) / P_{t-1}^{M,real} \right] .$$

B.3. Moment Conditions with Time Aggregation

B.3.1. Consumption Growth

In section 2.3, we define the quarterly consumption growth Δc_τ^Q as

$$\Delta c_\tau^Q \equiv \log \left(\frac{\sum_{j=0}^2 C_{t-j}}{\sum_{j=0}^2 C_{t-3-j}} \right). \quad (\text{B.28})$$

Rearranging terms we obtain

$$\begin{aligned} \Delta c_\tau^Q &= \log \left[\left(\frac{C_t}{C_{t-3}} \right) \left(\frac{1 + \frac{C_{t-1}}{C_t} + \frac{C_{t-2}}{C_{t-1}} \frac{C_{t-1}}{C_t}}{1 + \frac{C_{t-4}}{C_{t-3}} + \frac{C_{t-5}}{C_{t-4}} \frac{C_{t-4}}{C_{t-3}}} \right) \right] \\ &= \log \left(\frac{C_t}{C_{t-3}} \right) + \log \left(1 + \frac{C_{t-1}}{C_t} + \frac{C_{t-2}}{C_{t-1}} \frac{C_{t-1}}{C_t} \right) - \log \left(1 + \frac{C_{t-4}}{C_{t-3}} + \frac{C_{t-5}}{C_{t-4}} \frac{C_{t-4}}{C_{t-3}} \right) \\ &= \sum_{j=0}^2 \Delta c_{t-j} + \log [1 + \exp(-\Delta c_t) + \exp(-\Delta c_t) \exp(-\Delta c_{t-1})] \\ &\quad - \log [1 + \exp(-\Delta c_{t-3}) + \exp(-\Delta c_{t-3}) \exp(-\Delta c_{t-4})]. \end{aligned} \quad (\text{B.29})$$

Let $T_{c,1} = \log [1 + \exp(-\Delta c_t) + \exp(-\Delta c_t) \exp(-\Delta c_{t-1})]$. A first-order Taylor series expansion of $T_{c,1}$ yields

$$\begin{aligned} T_{c,1} &\approx \log [1 + \exp(-\Delta c_t^*) + \exp(-\Delta c_t^*) \exp(-\Delta c_{t-1}^*)] \\ &\quad + [1 + \exp(-\Delta c_t^*) + \exp(-\Delta c_t^*) \exp(-\Delta c_{t-1}^*)] \left\{ \left[-\exp(-\Delta c_t^*) \right. \right. \\ &\quad \left. \left. - \exp(-\Delta c_t) \exp(-\Delta c_{t-1}) \right] (\Delta c_t - \Delta c_t^*) \right. \\ &\quad \left. + \left[-\exp(-\Delta c_t) \exp(-\Delta c_{t-1}) \right] (\Delta c_{t-1} - \Delta c_{t-1}^*) \right\}. \end{aligned} \quad (\text{B.30})$$

Letting $\Delta c_t^* = \Delta c_{t-1}^* = 0$, we have

$$T_{c,1} \approx \log(3) + \left(-\frac{2}{3}\right) \Delta c_t + \left(-\frac{1}{3}\right) \Delta c_{t-1} . \quad (\text{B.31})$$

Let $T_{c,2} = \log[1 + \exp(-\Delta c_{t-3}) + \exp(-\Delta c_{t-3}) \exp(-\Delta c_{t-4})]$. Analogously, we can show that a first-order approximation of $T_{c,2}$, evaluated at $\Delta c_{t-3}^* = \Delta c_{t-4}^* = 0$, is given by

$$T_{c,2} \approx \log(3) + \left(-\frac{2}{3}\right) \Delta c_{t-3} + \left(-\frac{1}{3}\right) \Delta c_{t-4} . \quad (\text{B.32})$$

Substituting $T_{c,1}$ and $T_{c,2}$ in equation (B.29), and rewriting the approximated solution to Δc_τ^Q using the state variables and the innovations, we obtain

$$\begin{aligned} \Delta c_\tau^Q &\approx \sum_{j=1}^3 \left(\frac{j}{3} \Delta c_{t+1-j}\right) + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3}\right) \Delta c_{t-2-j}\right] \\ &= 3\mu + \left\{ \sum_{j=1}^3 \left(\frac{j}{3} x_{t-j}\right) + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3}\right) x_{t-3-j}\right] \right\} \\ &\quad + \left\{ \sum_{j=1}^3 \left(\frac{j}{3} \sigma_{t-j} \eta_{t+1-j}\right) + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3}\right) \sigma_{t-3-j} \eta_{t-2-j}\right] \right\} . \end{aligned} \quad (\text{B.33})$$

Given our assumptions on the processes of the state variables and the distribution of the innovations, we can use Equation (C.6) to derive the analytical expressions of the moment conditions associated with quarterly consumption growth:

- Unconditional Mean of Consumption Growth:

$$E(\Delta c_\tau^Q) = 3\mu ;$$

- Unconditional Variance of Consumption Growth:

$$\begin{aligned}
var(\Delta c_\tau^Q) &= E \left\{ [\Delta c_\tau^Q - E(\Delta c_\tau^Q)] [\Delta c_\tau^Q - E(\Delta c_\tau^Q)] \right\} \\
&= E \left\{ \left\{ \sum_{j=1}^3 \left(\frac{j}{3} x_{t-j} \right) + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) x_{t-3-j} \right] \right. \right. \\
&\quad \left. \left. + \sum_{j=1}^3 \left(\frac{j}{3} \sigma_{t-j} \eta_{t+1-j} \right) + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) \sigma_{t-3-j} \eta_{t-2-j} \right] \right\}^2 \right\} \\
&= \frac{1}{9(1-\rho^2)} (19 + 32\rho + 20\rho^2 + 8\rho^3 + 2\rho^4) (\varphi_e \sigma_0)^2 + \frac{19}{9} \sigma_0^2;
\end{aligned}$$

- Unconditional First Auto-Covariance of Consumption Growth:

$$\begin{aligned}
cov(\Delta c_\tau^Q, \Delta c_{\tau-1}^Q) &= E \left\{ [\Delta c_\tau^Q - E(\Delta c_\tau^Q)] [\Delta c_{\tau-1}^Q - E(\Delta c_{\tau-1}^Q)] \right\} \\
&= E \left\{ \left\{ \sum_{j=1}^3 \left(\frac{j}{3} x_{t-j} \right) + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) x_{t-3-j} \right] \right. \right. \\
&\quad \left. \left. + \sum_{j=1}^3 \left(\frac{j}{3} \sigma_{t-j} \eta_{t+1-j} \right) + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) \sigma_{t-3-j} \eta_{t-2-j} \right] \right\} \right. \\
&\quad \left. \left\{ \sum_{j=1}^3 \left(\frac{j}{3} x_{t-3-j} \right) + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) x_{t-6-j} \right] \right. \right. \\
&\quad \left. \left. + \sum_{j=1}^3 \left(\frac{j}{3} \sigma_{t-3-j} \eta_{t-2-j} \right) + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) \sigma_{t-6-j} \eta_{t-5-j} \right] \right\} \right\} \\
&= \frac{1}{9(1-\rho^2)} (4 + 11\rho + 16\rho^2 + 19\rho^3 + 16\rho^4 + 10\rho^5 \\
&\quad + 4\rho^6 + \rho^7) (\varphi_e \sigma_0)^2 + \frac{4}{9} \sigma_0^2;
\end{aligned}$$

B.3.2. Dividend Growth

According to our definition of the quasi-annual dividend growth

$$\begin{aligned}\Delta d_\tau^A &\equiv \log \left(\frac{D_\tau^Q}{D_{\tau-4}^Q} \right) \\ &\equiv \log \left(\frac{\sum_{j=0}^2 D_{t-j}}{\sum_{j=0}^2 D_{t-12-j}} \right),\end{aligned}\tag{B.34}$$

we rewrite equation (B.34) as

$$\begin{aligned}\Delta d_\tau^A &= \log \left[\left(\frac{D_t}{D_{t-12}} \right) \left(\frac{1 + \frac{D_{t-1}}{D_t} + \frac{D_{t-2}}{D_{t-1}} \frac{D_{t-1}}{D_t}}{1 + \frac{D_{t-13}}{D_{t-12}} + \frac{D_{t-14}}{D_{t-13}} \frac{D_{t-13}}{D_{t-12}}} \right) \right] \\ &= \log \left(\frac{D_t}{D_{t-12}} \right) + \log \left(1 + \frac{D_{t-1}}{D_t} + \frac{D_{t-2}}{D_{t-1}} \frac{D_{t-1}}{D_t} \right) - \log \left(1 + \frac{D_{t-13}}{D_{t-12}} + \frac{D_{t-14}}{D_{t-13}} \frac{D_{t-13}}{D_{t-12}} \right) \\ &= \sum_{j=0}^{11} \Delta d_{t-j} + \log [1 + \exp(-\Delta d_t) + \exp(-\Delta d_t) \exp(-\Delta d_{t-1})] \\ &\quad - \log [1 + \exp(-\Delta d_{t-12}) + \exp(-\Delta d_{t-12}) \exp(-\Delta d_{t-13})].\end{aligned}\tag{B.35}$$

Taking a first-order approximation, evaluated at $\Delta d_i^* = 0$ for $i = t, t-1, t-12$ and $t-13$, and then rewriting Δd_τ^A in terms of the state variables and the exogenous shocks, we obtain

$$\begin{aligned}\Delta d_\tau^A &\approx \sum_{j=1}^2 \left(\frac{j}{3} \Delta d_{t+1-j} \right) + \sum_{j=0}^9 (\Delta d_{t-2-j}) + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) \Delta d_{t-11-j} \right] \\ &= 12\mu_d + \phi \left\{ \sum_{j=1}^2 \left(\frac{j}{3} x_{t-j} \right) + \sum_{j=0}^{10} x_{t-3-j} + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) x_{t-12-j} \right] \right\} \\ &\quad + \varphi_d \left\{ \sum_{j=1}^2 \left(\frac{j}{3} \sigma_{t-j} \eta_{t+1-j} \right) + \sum_{j=0}^{10} (\sigma_{t-3-j} \eta_{t-2-j}) \right. \\ &\quad \left. + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) \sigma_{t-12-j} \eta_{t-11-j} \right] \right\}.\end{aligned}\tag{B.36}$$

Therefore, the associated moment conditions are:

- Unconditional Mean of Dividend Growth:

$$E \left(\Delta c_{\tau}^Q \right) = 12\mu_d ;$$

- Unconditional Variance of Dividend Growth:

$$\begin{aligned} var \left(\Delta d_{\tau}^A \right) &= E \left\{ \left[\Delta d_{\tau}^A - E \left(\Delta d_{\tau}^A \right) \right] \left[\Delta d_{\tau}^A - E \left(\Delta d_{\tau}^A \right) \right] \right\} \\ &= E \left\{ \left\{ \phi \left\{ \sum_{j=1}^2 \left(\frac{j}{3} x_{t-j} \right) + \sum_{j=0}^{10} x_{t-3-j} + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) x_{t-12-j} \right] \right\} \right. \right. \\ &\quad \left. \left. + \varphi_d \left\{ \sum_{j=1}^2 \left(\frac{j}{3} \sigma_{t-j} \eta_{t+1-j} \right) + \sum_{j=0}^{10} \left(\sigma_{t-3-j} \eta_{t-2-j} \right) \right. \right. \right. \\ &\quad \left. \left. \left. + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) \sigma_{t-12-j} \eta_{t-11-j} \right] \right\} \right\}^2 \right\} \\ &= \frac{\phi^2 (\varphi_e \sigma_0)^2}{(1-\rho^2)} \left\{ \frac{100}{9} + \frac{194}{9} \rho + \sum_{j=0}^8 2 (10-j) \rho^{2+j} + \frac{20}{9} \rho^{11} \right. \\ &\quad \left. + \sum_{j=1}^2 \left[\frac{20-6(j+1)}{9} \right] \rho^{11+j} \right\} + \frac{100}{9} (\varphi_d \sigma_0)^2 \end{aligned}$$

- Unconditional First Auto-Covariance of Dividend Growth:

$$\begin{aligned}
cov(\Delta d_\tau^A, \Delta d_{\tau-1}^A) &= E \left\{ [\Delta d_\tau^A - E(\Delta d_\tau^A)] [\Delta d_{\tau-1}^A - E(\Delta d_{\tau-1}^A)] \right\} \\
&= E \left\{ \left\{ \phi \left\{ \sum_{j=1}^2 \left(\frac{j}{3} x_{t-j} \right) + \sum_{j=0}^{10} x_{t-3-j} + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) x_{t-12-j} \right] \right\} \right. \right. \\
&\quad \left. \left. + \varphi_d \left\{ \sum_{j=1}^2 \left(\frac{j}{3} \sigma_{t-j} \eta_{t+1-j} \right) + \sum_{j=0}^{10} (\sigma_{t-3-j} \eta_{t-2-j}) \right. \right. \right. \\
&\quad \left. \left. \left. + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) \sigma_{t-12-j} \eta_{t-11-j} \right] \right\} \right\} \right. \\
&\quad \left. \left\{ \phi \left\{ \sum_{j=1}^2 \left(\frac{j}{3} x_{t-3-j} \right) + \sum_{j=0}^{10} x_{t-6-j} + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) x_{t-15-j} \right] \right\} \right. \right. \\
&\quad \left. \left. + \varphi_d \left\{ \sum_{j=1}^2 \left(\frac{j}{3} \sigma_{t-3-j} \eta_{t-2-j} \right) + \sum_{j=0}^{10} (\sigma_{t-6-j} \eta_{t-5-j}) \right. \right. \right. \\
&\quad \left. \left. \left. + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) \sigma_{t-15-j} \eta_{t-14-j} \right] \right\} \right\} \right\} \\
&= \frac{\phi^2 (\varphi_e \sigma_0)^2}{(1-\rho^2)} \left\{ \frac{4}{9} + \frac{4}{9} \rho + \sum_{j=0}^8 (2+j) \rho^{2+j} + \left(\frac{16}{9} + 9 \right) \rho^{11} \right. \\
&\quad \left. + \left(\frac{10}{9} + 10 \right) \rho^{12} + \left(\frac{16}{9} + 9 \right) \rho^{13} + \sum_{j=0}^8 (10-j) \rho^{14+j} \right. \\
&\quad \left. + \frac{10}{9} \rho^{23} + \sum_{j=1}^2 \left[\frac{10-3(j+1)}{9} \right] \rho^{23+j} \right\} + \frac{4}{9} (\varphi_d \sigma_0)^2 .
\end{aligned}$$

B.3.3. Price-Dividend Ratio

The logarithm of the quarterly price-dividend ratio $\Delta z_{d,\tau}^Q$ is defined as

$$\Delta z_{d,\tau}^Q \equiv \log \left(\frac{P_t}{\sum_{j=0}^2 D_{t-j}} \right). \quad (\text{B.37})$$

We rewrite equation (B.37) in the following way:

$$\begin{aligned} \Delta z_{d,\tau}^Q &= \log \left[\frac{P_{d,t}}{D_t \left(1 + \frac{D_{t-1}}{D_t} + \frac{D_{t-2}}{D_{t-1}} \frac{D_{t-1}}{D_t} \right)} \right] \\ &= \log \left(\frac{P_{d,t}}{D_t} \right) - \log \left(1 + \frac{D_{t-1}}{D_t} + \frac{D_{t-2}}{D_{t-1}} \frac{D_{t-1}}{D_t} \right) \\ &= z_{d,t} - \log [1 + \exp(-\Delta d_t) + \exp(-\Delta d_t) \exp(-\Delta d_{t-1})]. \end{aligned} \quad (\text{B.38})$$

Then, a first-order approximation of $\Delta z_{d,\tau}^Q$, evaluated at $\Delta d_t^* = \Delta d_{t-1}^* = 0$, suggests that

$$\Delta z_{d,\tau}^Q \approx z_{d,t} + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) \Delta d_{t+1-j} \right] - \log(3). \quad (\text{B.39})$$

Finally, plugging in the solution to $z_{d,t}$ and substituting the Δd 's using the state variables and the innovations, we find

$$\begin{aligned} \Delta z_{d,\tau}^Q &\approx z_{d,t} + \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) \Delta d_{t+1-j} \right] - \log(3) \\ &= [A_{0,d} + \mu_d - \log(3)] + \left\{ A_{1,d} x_t + \phi \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) x_{t-j} \right] \right\} \\ &\quad + A_{2,d} h_t + A_{3,d} \sigma_t^2 + A_{4,d} \sigma_{\lambda,t}^2 + \varphi_d \left\{ \sum_{j=1}^2 \left[\left(1 - \frac{j}{3} \right) \sigma_{t-j} u_{t+1-j} \right] \right\}. \end{aligned} \quad (\text{B.40})$$

Based on Equation (B.40), we are able to derive the analytical expressions of the following moment conditions:

- Unconditional Mean of Price-Dividend Ratio:

$$E\left(z_{d,\tau}^Q\right) = A_{0,d} + \mu_d - \log(3) + A_{3,d}\sigma_0^2 + A_{4,d}\sigma_\lambda^2;$$

- Unconditional Variance of Price-Dividend Ratio:

$$\begin{aligned} \text{var}\left(z_{d,\tau}^Q\right) &= E\left\{\left[z_{d,\tau}^Q - E\left(z_{d,\tau}^Q\right)\right]\left[z_{d,\tau}^Q - E\left(z_{d,\tau}^Q\right)\right]\right\} \\ &= E\left\{\left\{A_{1,d}x_t + \phi \sum_{j=1}^2 \left[\left(1 - \frac{j}{3}\right)x_{t-j}\right] + A_{2,d}h_t + A_{3,d}(\sigma_t^2 - \sigma_0^2) \right. \right. \\ &\quad \left. \left. + A_{4,d}(\sigma_{\lambda,t}^2 - \sigma_\lambda^2) + \varphi_d \sum_{j=1}^2 \left[\left(1 - \frac{j}{3}\right)\sigma_{t-j}u_{t+1-j}\right]\right\}^2\right\} \\ &= \frac{(\varphi_e\sigma_0)^2}{(1-\rho^2)} \left[(A_{1,d}^2 + \frac{5}{9}\phi^2) + \left(\frac{4}{3}\phi A_{1,d} + \frac{4}{9}\phi^2\right)\rho + \frac{2}{3}\phi A_{1,d}\rho^2 \right] \\ &\quad + \frac{5}{9}(\varphi_d\sigma)^2 + \left(\frac{A_{2,d}^2}{1-\rho_\lambda^2}\right)\sigma_\lambda^2 + \left(\frac{A_{3,d}^2}{1-\rho_1^2}\right)\sigma_w^2 + \left(\frac{A_{4,d}^2}{1-\rho_\lambda^2}\right)\sigma_\pi^2; \end{aligned}$$

- Unconditional First Auto-Covariance of Price-Dividend Ratio:

$$\begin{aligned}
cov(z_{d,\tau}^Q, z_{d,\tau-1}^Q) &= E \left\{ \left[z_{d,\tau}^Q - E(z_{d,\tau}^Q) \right] \left[z_{d,\tau-1}^Q - E(z_{d,\tau-1}^Q) \right] \right\} \\
&= E \left\{ \left\{ A_{1,d}x_t + \phi \sum_{j=1}^2 \left[\left(1 - \frac{j}{3}\right) x_{t-j} \right] + A_{2,d}h_t + A_{3,d}(\sigma_t^2 - \sigma_0^2) \right. \right. \\
&\quad \left. \left. + A_{4,d}(\sigma_{\lambda,t}^2 - \sigma_{\lambda}^2) + \varphi_d \sum_{j=1}^2 \left[\left(1 - \frac{j}{3}\right) \sigma_{t-j} u_{t+1-j} \right] \right\} \right. \\
&\quad \left. \left\{ A_{1,d}x_{t-3} + \phi \sum_{j=1}^2 \left[\left(1 - \frac{j}{3}\right) x_{t-3-j} \right] + A_{2,d}h_{t-3} + A_{3,d}(\sigma_{t-3}^2 - \sigma_0^2) \right. \right. \\
&\quad \left. \left. + A_{4,d}(\sigma_{\lambda,t-3}^2 - \sigma_{\lambda}^2) + \varphi_d \sum_{j=1}^2 \left[\left(1 - \frac{j}{3}\right) \sigma_{t-3-j} u_{t-2-j} \right] \right\} \right\} \\
&= \frac{(\varphi_e \sigma_0)^2}{(1-\rho^2)} \left[\frac{1}{3} \phi A_{1,d} \rho + \left(\frac{2}{3} \phi A_{1,d} + \frac{2}{9} \phi^2 \right) \rho^2 + \left(A_{1,d}^2 + \frac{5}{9} \phi^2 \right) \rho^3 \right. \\
&\quad \left. + \left(\frac{2}{3} \phi A_{1,d} + \frac{2}{9} \phi^2 \right) \rho^4 + \frac{1}{3} \phi A_{1,d} \rho^5 \right] + \left(\frac{A_{2,d}^2 \rho_{\lambda}^3}{1-\rho_{\lambda}^2} \right) \sigma_{\lambda}^2 \\
&\quad + \left(\frac{A_{3,d}^2 v_1^3}{1-v_1^2} \right) \sigma_w^2 + \left(\frac{A_{4,d}^2 v_{\lambda}^3}{1-v_{\lambda}^2} \right) \sigma_{\pi}^2.
\end{aligned}$$

B.3.4. Risk-Free Rate

According to our definition of the quarterly risk-free rate, it is straightforward to show that

$$\begin{aligned}
 r_{f,\tau}^Q &= \sum_{j=0}^2 r_{f,t-j} \\
 &= 3A_{0,rf} + A_{1,rf} \left(\sum_{j=0}^2 x_{t-j} \right) + A_{2,rf} \left(\sum_{j=0}^2 h_{t-j} \right) \\
 &\quad + A_{3,rf} \left(\sum_{j=0}^2 \sigma_{t-j}^2 \right) + A_{4,rf} \left(\sum_{j=0}^2 \sigma_{\lambda,t-j}^2 \right).
 \end{aligned} \tag{B.41}$$

Then, the first and the second moments of $r_{f,\tau}^Q$ are given by:

- Unconditional Mean of Risk-Free Rate:

$$E \left(r_{f,\tau}^Q \right) = 3 \left(A_{0,rf} + A_{3,rf} \sigma_0^2 + A_{4,rf} \sigma_\lambda^2 \right) \quad ;$$

- Unconditional Variance of Risk-Free Rate:

$$\begin{aligned}
 var \left(r_{f,\tau}^Q \right) &= E \left\{ \left[r_{f,\tau}^Q - E \left(r_{f,\tau}^Q \right) \right] \left[r_{f,\tau}^Q - E \left(r_{f,\tau}^Q \right) \right] \right\} \\
 &= E \left\{ \left\{ A_{1,rf} \left(\sum_{j=0}^2 x_{t-j} \right) + A_{2,rf} \left(\sum_{j=0}^2 h_{t-j} \right) \right. \right. \\
 &\quad \left. \left. A_{3,rf} \left[\sum_{j=0}^2 \left(\sigma_{t-j}^2 - \sigma_0^2 \right) \right] + A_{4,rf} \left[\sum_{j=0}^2 \left(\sigma_{\lambda,t-j}^2 - \sigma_\lambda^2 \right) \right] \right\}^2 \right\} \\
 &= \frac{A_{1,rf}^2 (\varphi_e \sigma_0)^2}{(1-\rho^2)} (3 + 4\rho + 2\rho^2) + \frac{A_{2,rf}^2 \sigma_\lambda^2}{(1-\rho_\lambda^2)} (3 + 4\rho_\lambda + 2\rho_\lambda^2) \\
 &\quad + \frac{A_{3,rf}^2 \sigma_w^2}{(1-v_1^2)} (3 + 4v_1 + 2v_1^2) + \frac{A_{4,rf}^2 \sigma_\pi^2}{(1-v_\lambda^2)} (3 + 4v_\lambda + 2v_\lambda^2) .
 \end{aligned}$$

B.3.5. Equity Return

We define the quarterly return on the equity asset in period $\tau + 1$ as the summation of the logarithm of the asset's gross returns over the current and the past two months:

$$r_{d,\tau+1}^Q = \sum_{j=0}^2 r_{d,t+1+j} \ . \quad (\text{B.42})$$

Note that the equation

$$r_{d,t+1+j} = k_{0,d} + k_{1,d} z_{d,t+1+j} - z_{d,t+j} + \Delta d_{t+1+j} \quad (\text{B.43})$$

holds for all $j \in N$. Then, substitute $z_{d,t+j}$ and rewrite equation (B.42) in terms of the state variables and the exogenous processes. After a bit arithmetics, it can be shown that

$$\begin{aligned}
r_{d,\tau+1}^Q &= 3 [k_{0,d} + A_{0,d} (k_{1,d} - 1) + \mu_d + A_{3,d} (k_{1,d} - 1) \sigma_0^2 + A_{4,d} (k_{1,d} - 1) \sigma_\lambda^2] \\
&+ \frac{1}{\psi} \left(\sum_{j=0}^2 x_{t+j} \right) - \rho_\lambda \left(\sum_{j=0}^2 h_{t+j} \right) + A_{3,d} (k_{1,d} v_1 - 1) \left[\sum_{j=0}^2 (\sigma_{t+j}^2 - \sigma_0^2) \right] \\
&+ A_{4,d} (k_{1,d} v_\lambda - 1) \left[\sum_{j=0}^2 (\sigma_{\lambda,t+j}^2 - \sigma_\lambda^2) \right] + k_{1,d} A_{1,d} \varphi_e \left(\sum_{j=0}^2 \sigma_{t+j} e_{t+1+j} \right) \\
&k_{1,d} A_{2,d} \left(\sum_{j=0}^2 \sigma_{\lambda,t+j} \varepsilon_{t+1+j} \right) + k_{1,d} A_{3,d} \sigma_w \left(\sum_{j=0}^2 w_{t+1+j} \right) \\
&+ k_{1,d} A_{4,d} \sigma_\pi \left(\sum_{j=0}^2 \pi_{t+1+j} \right) + \varphi_d \left(\sum_{j=0}^2 \sigma_{t+j} u_{t+1+j} \right) \\
\\
&= 3 [k_{0,d} + A_{0,d} (k_{1,d} - 1) + \mu_d + A_{3,d} (k_{1,d} - 1) \sigma_0^2 + A_{4,d} (k_{1,d} - 1) \sigma_\lambda^2] \\
&+ \frac{1}{\psi} \left(\sum_{j=0}^2 \rho^j \right) x_t - \rho_\lambda \left(\sum_{j=0}^2 \rho_\lambda^j \right) h_t + A_{3,d} (k_{1,d} v_1 - 1) \sum_{j=0}^2 (v_1^j) (\sigma_t^2 - \sigma_0^2) \\
&+ A_{4,d} (k_{1,d} v_\lambda - 1) \sum_{j=0}^2 (v_\lambda^j) (\sigma_{\lambda,t}^2 - \sigma_\lambda^2) \\
&+ \sum_{j=0}^1 \left\{ \left[k_{1,d} A_{1,d} + \frac{1}{\psi} (1 + \rho)^{1-j} \right] \varphi_e \sigma_{t+j} e_{t+1+j} \right\} + k_{1,d} A_{1,d} \sigma_{t+2} e_{t+3} \\
&+ \sum_{j=0}^1 \left\{ \left[k_{1,d} A_{2,d} - \rho_\lambda (1 + \rho_\lambda)^{1-j} \right] \sigma_{\lambda,t+j} \varepsilon_{t+1+j} \right\} + k_{1,d} A_{2,d} \sigma_{\lambda,t+2} \varepsilon_{t+3} \\
&+ \sum_{j=0}^1 \left\{ \left[k_{1,d} A_{3,d} + A_{3,d} (k_{1,d} v_1 - 1) (1 + v_1)^{1-j} \right] \sigma_w w_{t+1+j} \right\} + k_{1,d} A_{3,d} \sigma_w w_{t+3} \\
&+ \sum_{j=0}^1 \left\{ \left[k_{1,d} A_{4,d} + A_{4,d} (k_{1,d} v_\lambda - 1) (1 + v_\lambda)^{1-j} \right] \sigma_\pi \pi_{t+1+j} \right\} + k_{1,d} A_{4,d} \sigma_\pi \pi_{t+3} \\
&+ \varphi_d \left(\sum_{j=0}^2 \sigma_{t+j} u_{t+1+j} \right).
\end{aligned} \tag{B.44}$$

Equation (B.44) implies that first and second moments of the equity return are:

- Unconditional Mean of Equity Return:

$$E\left(r_{d,\tau+1}^Q\right) = 3\left[k_{0,d} + A_{0,d}(k_{1,d} - 1) + \mu_d + A_{3,d}(k_{1,d} - 1)\sigma_0^2 + A_{4,d}(k_{1,d} - 1)\sigma_\lambda^2\right] \quad ;$$

- Unconditional Variance of Equity Return:

$$\begin{aligned} var\left(r_{d,\tau+1}^Q\right) &= E\left\{\left[r_{d,\tau+1}^Q - E\left(r_{d,\tau+1}^Q\right)\right]\left[r_{d,\tau+1}^Q - E\left(r_{d,\tau+1}^Q\right)\right]\right\} \\ &= \left[\frac{(\varphi_e\sigma_0)^2}{\psi^2(1-\rho^2)}\right]\left[\left(\sum_{j=0}^2\rho^j\right)\right]^2 + \left[\frac{\rho_\lambda^2\sigma_\lambda^2}{(1-\rho_\lambda^2)}\right]\left[\left(\sum_{j=0}^2\rho_\lambda^j\right)\right]^2 + 3(\varphi_d\sigma_0)^2 \\ &\quad + \left(\frac{\sigma_w^2}{1-v_1^2}\right)\left[A_{3,d}(k_{1,d}v_1 - 1)\sum_{j=0}^2(v_1^j)\right]^2 \\ &\quad + \left(\frac{\sigma_\pi^2}{1-v_\lambda^2}\right)\left[A_{4,d}(k_{1,d}v_\lambda - 1)\sum_{j=0}^2(v_\lambda^j)\right]^2 \\ &\quad + \left\{\sum_{j=0}^1\left[k_{1,d}A_{1,d} + \frac{1}{\psi}(1+\rho)^{1-j}\right]^2 + (k_{1,d}A_{1,d})^2\right\}(\varphi_e\sigma_0)^2 \\ &\quad + \left\{\sum_{j=0}^1\left[k_{1,d}A_{2,d} - \rho_\lambda(1+\rho_\lambda)^{1-j}\right]^2 + (k_{1,d}A_{2,d})^2\right\}\sigma_\lambda^2 \\ &\quad + \left\{\sum_{j=0}^1\left[k_{1,d}A_{3,d} + A_{3,d}(k_{1,d}v_1 - 1)(1+v_1)^{1-j}\right]^2 + (k_{1,d}A_{3,d})^2\right\}\sigma_w^2 \\ &\quad + \left\{\sum_{j=0}^1\left[k_{1,d}A_{4,d} + A_{4,d}(k_{1,d}v_\lambda - 1)(1+v_\lambda)^{1-j}\right]^2 + (k_{1,d}A_{4,d})^2\right\}\sigma_\pi^2. \end{aligned}$$

B.3.6. Equity Premium

Based on equation (B.41), we know that the risk-free rate at time $\tau + 1$ satisfies

$$\begin{aligned}
r_{f,\tau+1}^Q &= 3A_{0,rf} + A_{1,rf} \left(\sum_{j=0}^2 x_{t+3-j} \right) + A_{2,rf} \left(\sum_{j=0}^2 h_{t+3-j} \right) \\
&\quad + A_{3,rf} \left(\sum_{j=0}^2 \sigma_{t+3-j}^2 \right) + A_{4,rf} \left(\sum_{j=0}^2 \sigma_{\lambda,t+3-j}^2 \right) . \\
&= 3(A_{0,rf} + A_{3,rf}\sigma_0^2 + A_{4,rf}\sigma_\lambda^2) + \rho A_{1,rf} \left(\sum_{j=0}^2 x_{t+j} \right) \\
&\quad + \rho_\lambda A_{2,rf} \left(\sum_{j=0}^2 h_{t+j} \right) + v_1 A_{3,rf} \left(\sum_{j=0}^2 \sigma_{t+j}^2 \right) + v_\lambda A_{4,rf} \left(\sum_{j=0}^2 \sigma_{\lambda,t+j}^2 \right) \\
&\quad + A_{1,rf} \varphi_e \sum_{j=0}^2 (\sigma_{t+j} e_{t+1+j}) + A_{2,rf} \sum_{j=0}^2 (\sigma_{\lambda,t+j} \varepsilon_{t+1+j}) \\
&\quad + A_{3,rf} \sigma_w \sum_{j=0}^2 (w_{t+1+j}) + A_{4,rf} \sigma_\pi \sum_{j=0}^2 (\pi_{t+1+j}) .
\end{aligned} \tag{B.45}$$

Therefore, combining equations (B.44) and (B.45), we solve for the realized quarterly equity premium, $ep_{\tau+1}$, which is given by

$$\begin{aligned}
ep_{\tau+1} &\equiv r_{d,\tau+1}^Q - r_{f,\tau+1}^Q \\
&= 3 \left\{ k_{0,d} + A_{0,d} (k_{1,d} - 1) + \mu_d - A_{0,rf} + [A_{3,d} (k_{1,d} - 1) - A_{3,rf}] \sigma_0^2 \right. \\
&\quad \left. + [A_{4,d} (k_{1,d} - 1) - A_{4,rf}] \sigma_\lambda^2 \right\} + \left(\frac{1}{\psi} - \rho A_{1,rf} \right) \left(\sum_{j=0}^2 x_{t+j} \right) \\
&\quad + (-\rho_\lambda - \rho_\lambda A_{2,rf}) \left(\sum_{j=0}^2 h_{t+j} \right) + [A_{3,d} (k_{1,d} v_1 - 1) - v_1 A_{3,rf}] \left(\sum_{j=0}^2 \sigma_{t+j}^2 \right) \\
&\quad + [A_{4,d} (k_{1,d} v_\lambda - 1) - v_\lambda A_{4,rf}] \left(\sum_{j=0}^2 \sigma_{\lambda,t+j}^2 \right) + (k_{1,d} A_{1,d} - A_{1,rf}) \varphi_e \sum_{j=0}^2 (\sigma_{t+j} e_{t+1+j}) \\
&\quad + (k_{1,d} A_{2,d} - A_{2,rf}) \sum_{j=0}^2 (\sigma_{\lambda,t+j} \varepsilon_{t+1+j}) + (k_{1,d} A_{3,d} - A_{3,rf}) \sigma_w \sum_{j=0}^2 (w_{t+1+j}) \\
&\quad + (k_{1,d} A_{4,d} - A_{4,rf}) \sigma_\pi \sum_{j=0}^2 (\pi_{t+1+j}) + \varphi_d \left(\sum_{j=0}^2 \sigma_{t+j} u_{t+1+j} \right) \\
&= 3s_0 + s_1 \left(\sum_{j=0}^2 x_{t+j} \right) + s_2 \left(\sum_{j=0}^2 h_{t+j} \right) + s_3 \left(\sum_{j=0}^2 \sigma_{t+j}^2 \right) + s_4 \left(\sum_{j=0}^2 \sigma_{\lambda,t+j}^2 \right) \\
&\quad + s_5 \varphi_e \sum_{j=0}^2 (\sigma_{t+j} e_{t+1+j}) + s_6 \sum_{j=0}^2 (\sigma_{\lambda,t+j} \varepsilon_{t+1+j}) + s_7 \sigma_w \sum_{j=0}^2 (w_{t+1+j}) \\
&\quad + s_8 \sigma_\pi \sum_{j=0}^2 (\pi_{t+1+j}) + \varphi_d \left(\sum_{j=0}^2 \sigma_{t+j} u_{t+1+j} \right) \\
&= 3s_0 + s_1 \left(\sum_{j=0}^2 \rho^j \right) x_t + s_2 \left(\sum_{j=0}^2 \rho_\lambda^j \right) h_t \\
&\quad + s_3 \sum_{j=0}^2 (v_1^j) (\sigma_t^2 - \sigma_0^2) + s_4 \sum_{j=0}^2 (v_\lambda^j) (\sigma_{\lambda,t}^2 - \sigma_\lambda^2) \\
&\quad + \sum_{j=0}^1 \left\{ \left[s_5 + (1 + \rho)^{1-j} s_1 \right] \varphi_e \sigma_{t+j} e_{t+1+j} \right\} + s_5 \sigma_{t+2} e_{t+3} \\
&\quad + \sum_{j=0}^1 \left\{ \left[s_6 + (1 + \rho_\lambda)^{1-j} s_2 \right] \sigma_{\lambda,t+j} \varepsilon_{t+1+j} \right\} + s_6 \sigma_{\lambda,t+2} \varepsilon_{t+3} \\
&\quad + \sum_{j=0}^1 \left\{ \left[s_7 + (1 + v_1)^{1-j} s_3 \right] \sigma_w w_{t+1+j} \right\} + s_7 \sigma_w w_{t+3} \\
&\quad + \sum_{j=0}^1 \left\{ \left[s_8 + (1 + v_\lambda)^{1-j} s_4 \right] \sigma_\pi \pi_{t+1+j} \right\} + s_8 \sigma_w \pi_{t+3} + \varphi_d \left(\sum_{j=0}^2 \sigma_{t+j} u_{t+1+j} \right). \tag{B.46}
\end{aligned}$$

Equation (B.46) implies that the first and second moments of the equity premium are:

- Unconditional Mean of Equity Premium:

$$E(ep_{\tau+1}) = 3s_0 \quad ;$$

- Unconditional Variance of Equity Premium:

$$\begin{aligned} \text{var}(ep_{\tau+1}) &= E\{[ep_{\tau+1} - E(ep_{\tau+1})][ep_{\tau+1} - E(ep_{\tau+1})]\} \\ &= \frac{s_1^2(\varphi_e\sigma_0)^2}{(1-\rho^2)} \left[\left(\sum_{j=0}^2 \rho^j \right) \right]^2 + \frac{s_2^2\sigma_\lambda^2}{(1-\rho_\lambda^2)} \left[\left(\sum_{j=0}^2 \rho_\lambda^j \right) \right]^2 + 3(\varphi_d\sigma_0)^2 \\ &\quad + \frac{s_3^2\sigma_w^2}{(1-v_1^2)} \left[\sum_{j=0}^2 (v_1^j) \right]^2 + \frac{s_4^2\sigma_\pi^2}{(1-v_\lambda^2)} \left[\sum_{j=0}^2 (v_\lambda^j) \right]^2 \\ &\quad + \left\{ \sum_{j=0}^1 \left[s_5 + (1+\rho)^{1-j} s_1 \right]^2 + s_5^2 \right\} (\varphi_e\sigma_0)^2 \\ &\quad + \left\{ \sum_{j=0}^1 \left[s_6 + (1+\rho)^{1-j} s_2 \right]^2 + s_6^2 \right\} \sigma_\lambda^2 \\ &\quad + \left\{ \sum_{j=0}^1 \left[s_7 + (1+v_1)^{1-j} s_3 \right]^2 + s_7^2 \right\} \sigma_w^2 \\ &\quad + \left\{ \sum_{j=0}^1 \left[s_8 + (1+v_\lambda)^{1-j} s_4 \right]^2 + s_8^2 \right\} \sigma_\pi^2 . \end{aligned}$$

B.3.7. Other Moments: Predictability and Weak Correlation

- Unconditional Covariance between Δc_τ^Q and $z_{d,\tau-1}^Q$:

$$\begin{aligned}
cov\left(\Delta c_\tau^Q, z_{d,\tau-1}^Q\right) &= E\left\{\left[\Delta c_\tau^Q - E\left(\Delta c_\tau^Q\right)\right]\left[z_{d,\tau-1}^Q - E\left(z_{d,\tau-1}^Q\right)\right]\right\} \\
&= (\varphi_e \sigma_0)^2 \left\{ \left(\frac{1}{1-\rho^2}\right) \left[\left(\sum_{j=1}^3 \frac{j}{3} \rho^{5-j}\right) \right. \right. \\
&\quad \left. \left. + \left(\sum_{j=1}^2 \frac{j}{3} \rho^{j-1}\right) \right] (A_{1,d} \rho^2 + \frac{2}{3} \phi \rho + \frac{1}{3} \phi) \right. \\
&\quad \left. + \left[\left(\sum_{j=1}^3 \frac{j}{3} \rho^{4-j}\right) + \frac{2}{3}\right] (A_{1,d} \rho + \frac{2}{3} \phi) + \left[\left(\sum_{j=1}^3 \frac{j}{3} \rho^{3-j}\right) A_{1,d}\right] \right\};
\end{aligned}$$

- Unconditional Covariance between Δc_τ^Q and $z_{d,\tau-4}^Q$:

$$\begin{aligned}
cov\left(\Delta c_\tau^Q, z_{d,\tau-4}^Q\right) &= E\left\{\left[\Delta c_\tau^Q - E\left(\Delta c_\tau^Q\right)\right]\left[z_{d,\tau-4}^Q - E\left(z_{d,\tau-4}^Q\right)\right]\right\} \\
&= \frac{(\varphi_e \sigma_0)^2}{(1-\rho^2)} \left[\left(\sum_{j=1}^3 \frac{j}{3} \rho^{5-j}\right) + \left(\sum_{j=1}^2 \frac{j}{3} \rho^{j-1}\right) \right] (A_{1,d} \rho^7 + \frac{2}{3} \phi \rho^8 + \frac{1}{3} \phi \rho^9);
\end{aligned}$$

- Unconditional Covariance between $r_{d,\tau}^Q$ and $z_{d,\tau-1}^Q$:

$$\begin{aligned}
cov\left(r_{d,\tau}^Q, z_{d,\tau-1}^Q\right) &= E\left\{\left[r_{d,\tau}^Q - E\left(r_{d,\tau}^Q\right)\right]\left[z_{d,\tau-1}^Q - E\left(z_{d,\tau-1}^Q\right)\right]\right\} \\
&= \frac{(\varphi_e \sigma_0)^2}{\psi} \left(\sum_{j=0}^2 \rho^j\right) \left[\left(\frac{\rho^2}{1-\rho^2}\right) (A_{1,d} \rho^2 + \frac{2}{3} \phi \rho + \frac{1}{3} \phi) \right. \\
&\quad \left. + \rho (A_{1,d} \rho + \frac{2}{3} \phi) + A_{1,d} \right] + \left(\frac{-\rho_\lambda A_{2,d} \sigma_\lambda^2}{1-\rho_\lambda^2}\right) \left(\sum_{j=0}^2 \rho_\lambda^j\right) \\
&\quad + \left[\frac{A_{3,d}^2 (k_{1,d} v_1 - 1) \sigma_w^2}{1-v_1^2}\right] \left(\sum_{j=0}^2 v_1^j\right) + \left[\frac{A_{4,d}^2 (k_{1,d} v_\lambda - 1) \sigma_\pi^2}{1-v_\lambda^2}\right] \left(\sum_{j=0}^2 v_\lambda^j\right);
\end{aligned}$$

- Unconditional Covariance between $r_{d,\tau}^Q$ and $z_{d,\tau-4}^Q$:

$$\begin{aligned}
cov\left(r_{d,\tau}^Q, z_{d,\tau-4}^Q\right) &= E\left\{\left[r_{d,\tau}^Q - E\left(r_{d,\tau}^Q\right)\right]\left[z_{d,\tau-4}^Q - E\left(z_{d,\tau-4}^Q\right)\right]\right\} \\
&= \frac{(\varphi_e \sigma_0)^2 \rho^2}{\psi(1-\rho^2)} \left(\sum_{j=0}^2 \rho^j\right) \left(A_{1,d} \rho^7 + \frac{2}{3} \phi \rho^8 + \frac{1}{3} \phi \rho^9\right) \\
&\quad + \left(\frac{-\rho \lambda A_{2,d} \sigma_\lambda^2 \rho_\lambda^9}{1-\rho_\lambda^2}\right) \left(\sum_{j=0}^2 \rho_\lambda^j\right) + \left[\frac{A_{3,d}^2 (k_{1,d} v_1 - 1) v_1^9 \sigma_w^2}{1-v_1^2}\right] \left(\sum_{j=0}^2 v_1^j\right) \\
&\quad + \left[\frac{A_{4,d}^2 (k_{1,d} v_\lambda - 1) v_\lambda^9 \sigma_\pi^2}{1-v_\lambda^2}\right] \left(\sum_{j=0}^2 v_\lambda^j\right) ;
\end{aligned}$$

- Unconditional Covariance between $r_{d,\tau}^Q$ and Δc_τ^Q :

$$\begin{aligned}
cov\left(r_{d,\tau}^Q, \Delta c_\tau^Q\right) &= E\left\{\left[r_{d,\tau}^Q - E\left(r_{d,\tau}^Q\right)\right]\left[\Delta c_\tau^Q - E\left(\Delta c_\tau^Q\right)\right]\right\} \\
&= \frac{(\varphi_e \sigma_0)^2}{\psi} \left(\sum_{j=0}^2 \rho^j\right) \left\{\left(\frac{\rho^2}{1-\rho^2}\right) \left[\left(\sum_{j=1}^3 \frac{j}{3} \rho^{5-j}\right) + \left(\sum_{j=1}^2 \frac{j}{3} \rho^{j-1}\right)\right]\right. \\
&\quad \left.+ \rho \left[\left(\sum_{j=1}^3 \frac{j}{3} \rho^{4-j}\right) + \frac{2}{3}\right] + \left(\sum_{j=1}^3 \frac{j}{3} \rho^{3-j}\right)\right\} \\
&\quad (\varphi_e \sigma_0)^2 \left\{\left[k_{1,d} A_{1,d} + \frac{1}{\psi} (1 + \rho)\right] \left(\frac{1}{3} \rho + \frac{2}{3}\right) + \frac{1}{3} \left[k_{1,d} A_{1,d} + \frac{1}{\psi}\right]\right\} ;
\end{aligned}$$

- Unconditional Covariance between $r_{d,\tau}^Q$ and $\Delta c_{\tau-1}^Q$:

$$\begin{aligned}
cov\left(r_{d,\tau}^Q, \Delta c_{\tau-1}^Q\right) &= E\left\{\left[r_{d,\tau}^Q - E\left(r_{d,\tau}^Q\right)\right]\left[\Delta c_{\tau-1}^Q - E\left(\Delta c_{\tau-1}^Q\right)\right]\right\} \\
&= \frac{(\varphi_e \sigma_0)^2}{\psi} \left(\sum_{j=0}^2 \rho^j\right) \left\{\left(\frac{\rho^5}{1-\rho^2}\right) \left[\left(\sum_{j=1}^3 \frac{j}{3} \rho^{5-j}\right) + \left(\sum_{j=1}^2 \frac{j}{3} \rho^{j-1}\right)\right]\right. \\
&\quad \left.+ \rho^4 \left[\left(\sum_{j=1}^3 \frac{j}{3} \rho^{4-j}\right) + \frac{2}{3}\right] + \rho^3 \left(\sum_{j=1}^3 \frac{j}{3} \rho^{3-j}\right) + \rho^2 \left(\sum_{j=1}^2 \frac{j}{3} \rho^{2-j}\right) + \frac{1}{3} \rho\right\} .
\end{aligned}$$

B.4. Supplemental Figures

Figure B.1. Filtered States - Baseline

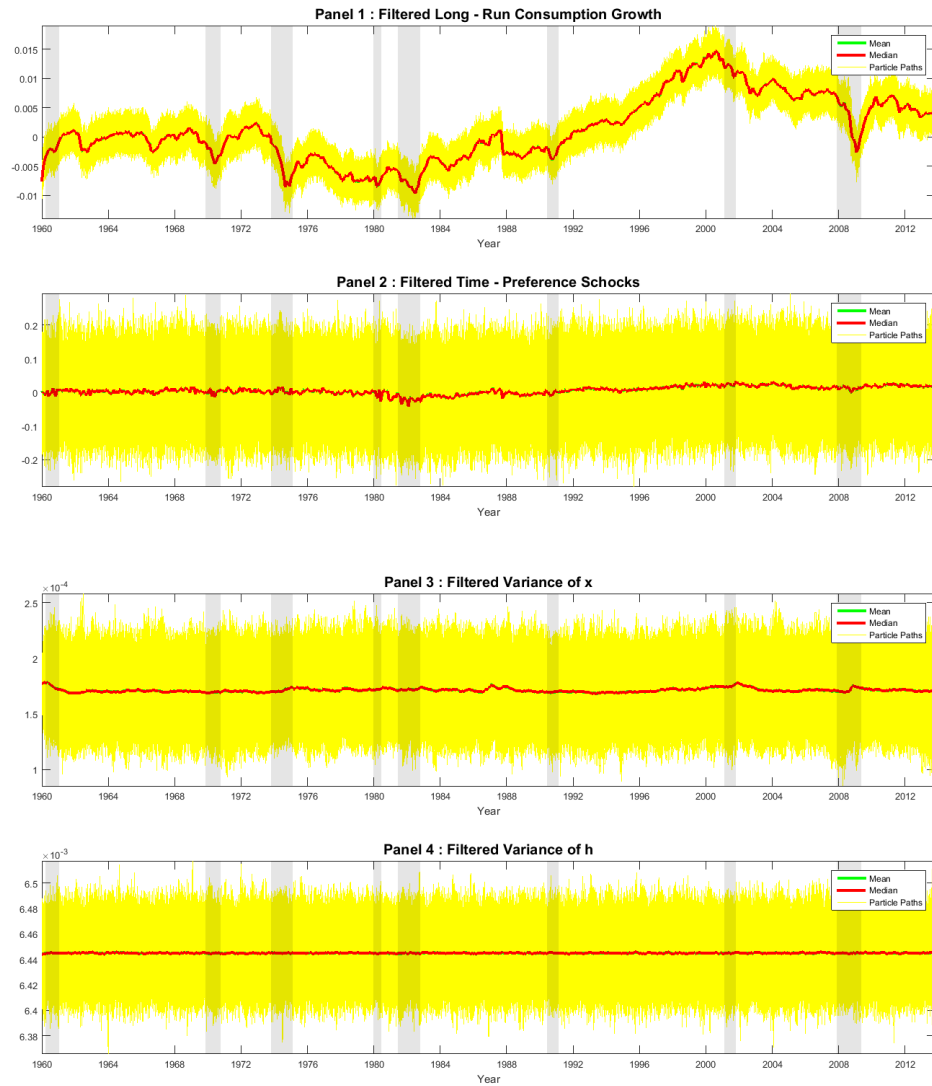


Figure B.2. Filtered and Smoothed States - LRC Only

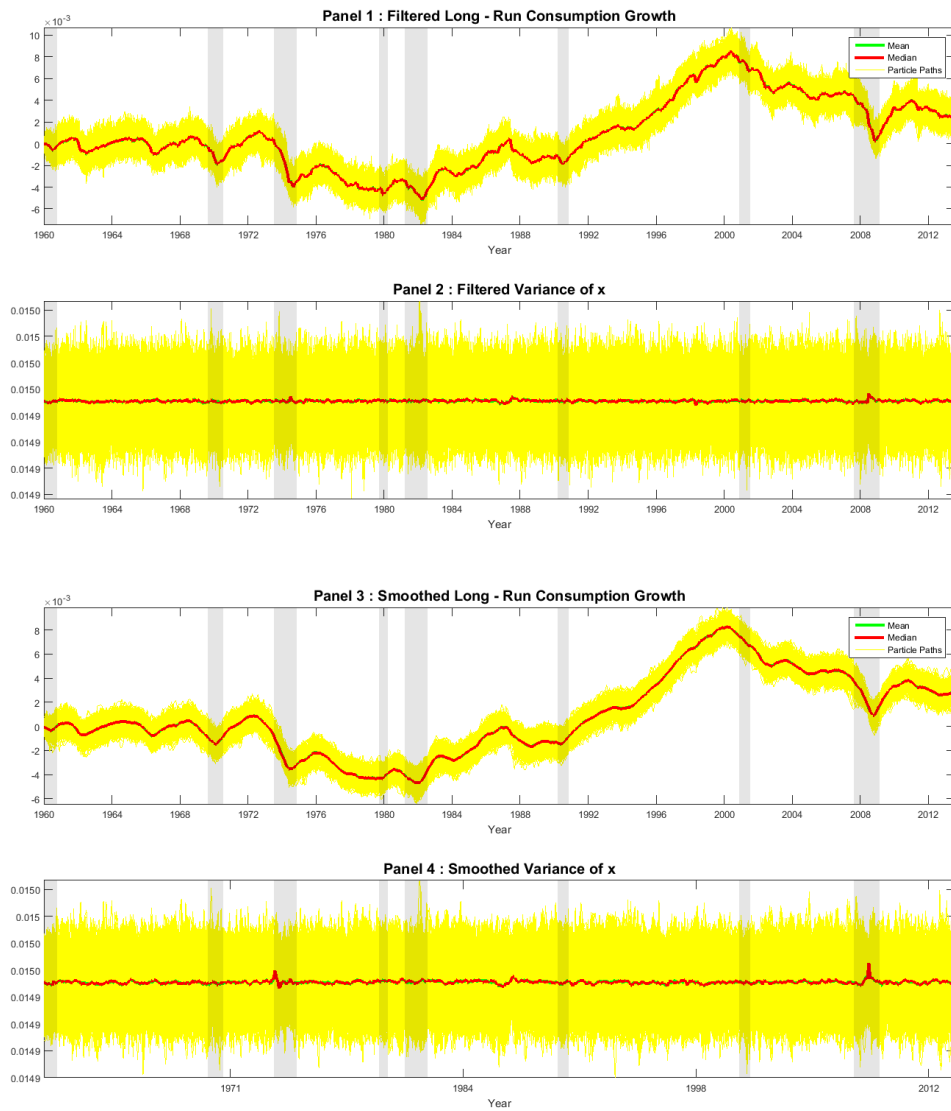
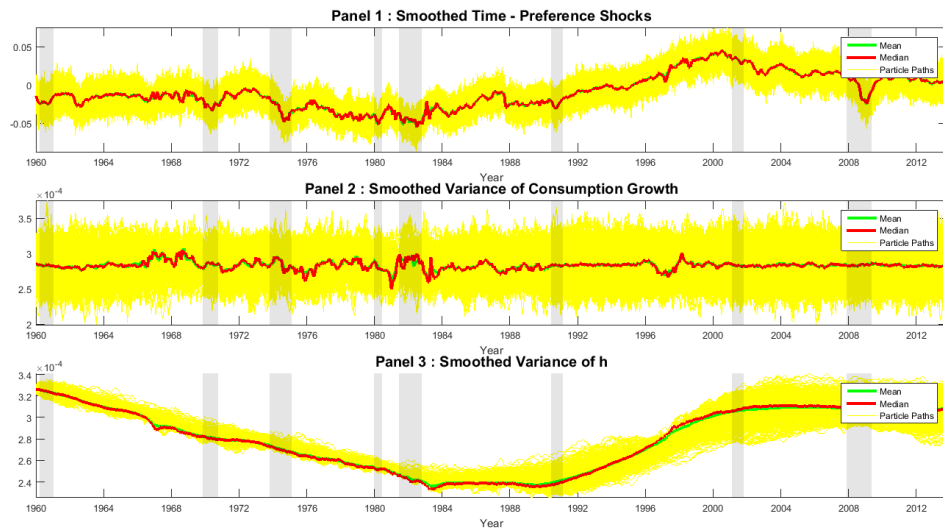


Figure B.3. Smoothed States - TPS Only - Persistent Time-Preference Shocks



APPENDIX C

Appendix of Chapter 3

C.1. Assessing The Credibility Of The Budget Sequestration Cuts With A Bayesian Approach

This section exploits the Bayesian Markov Chain Monte Carlo (MCMC) approach to estimate the parameter vector ψ , which governs the spending cuts scenarios.¹ Let $p(Y^T|\psi)$ and $\pi(\psi)$ denote the likelihood function and the prior distribution of the parameters, respectively. It can be shown that the posterior distribution of ψ , $\pi(\psi|Y^T)$, satisfies

$$\pi(\psi|Y^T) \propto p(Y^T|\psi) \pi(\psi) .$$

Given that the posterior distribution is not analytically tractable, this study employs a Random Walk Metropolis-Hastings (RW-MH) sampler to generate draws from the proposed distribution. The RW-MH sampling algorithm proceeds as follows:

Step 1. Start from $\psi^{(0)} = [\psi_0^{(0)}, \psi_1^{(0)}]' = [0.5, 0.5]'$, and use a symmetric random walk proposal g ;

Step 2. For $i = 1$, draw $\epsilon^{(i)} \sim g$ and set $\psi^c = \psi^{(i-1)} + \epsilon^{(i)}$;

Step 3. Compute the acceptance probability for the candidate draw:

$$\alpha(\psi^c|\psi^{(i-1)}) = \min \left\{ \frac{p(Y^T|\psi^c) \pi(\psi^c)}{p(Y^T|\psi^{(i-1)}) \pi(\psi^{(i-1)})} \frac{g(\psi^{(i-1)} - \psi^c)}{g(\psi^{(i-1)} - \psi^c)}, 1 \right\} = \min \left\{ \frac{p(Y^T|\psi^c) \pi(\psi^c)}{p(Y^T|\psi^{(i-1)}) \pi(\psi^{(i-1)})}, 1 \right\} ;$$

Step 4. Set $\psi^{(i)} = \psi^c$ with probability $\alpha(\psi^c|\psi^{(i-1)})$, and $\psi^{(i)} = \psi^{(i-1)}$ with probability $1 - \alpha(\psi^c|\psi^{(i-1)})$;

¹As indicated in section 3.4.2.3, $\psi = [\psi_0, \psi_1]'$.

Step 5. Repeat Step 2 - 4 for $i = 2, 3, \dots, N$.

In this application, the prior distribution of ψ_0 and ψ_1 is assumed to be $U(0, 1)$. Given these uninformative or diffuse priors, the posterior distribution is principally determined by the likelihood function.² In addition, this study assumes that the symmetric random walk proposal g is bivariate Gaussian with diagonal variance-covariance matrix Σ_g , where the standard deviation of ψ_0 is set at two-thirds of the standard deviation of ψ_1 . Σ_g is scaled to ensure that the acceptance rate is around 50%.³

Table C.1 and Figure C.1 - C.2 report the posterior distribution of ψ_0 and ψ_1 based on 80,000 draws after a burn-in period of 20,000 draws. For ψ_0 , which governs the credibility of the spending cut in 2013, the posterior distribution is concentrated at the high end of the zero-one interval. While the posterior mean is 0.61, rounding the accepted draws at the 2nd digit yields posterior mode that is equal to 0.97, which serves as strong empirical evidence that the announced spending cuts in 2013 are highly credible. In contrast, the data seems to provide very limited information to update the prior distribution of ψ_1 . The posterior distribution of ψ_1 seems to be close to uniform with estimated posterior mean 0.49. Given the limited evidence inherent to an event-study approach as the one adopted for this paper, it is rather remarkable that an uninformative prior distribution shifts the mode of the posterior distribution to 0.07. This feature of the posterior distribution, along with its general tendency to increase the frequency of the parameter ψ_1 at its value declines, relative to the flat frequencies of the prior, suggests that a Bayesian inference approach to the evidence favors rather unambiguously the low credibility spending cuts scenarios.

²Given the event-study nature of this paper, the likelihood function used to compute the acceptance probability incorporates exclusively the likelihood of the data in 2013.

³Computing the inverse Hessian of the logarithm of the likelihood in 2013, evaluated at $\psi_0 = \psi_1 = 0.5$, suggests that the standard deviation of ψ_0 is smaller than that of ψ_1 . However, estimation results are robust to other specifications on Σ_g , such as (1) $std(\psi_0) = std(\psi_1)$; and (2) $std(\psi_0) = \frac{3}{2}std(\psi_1)$.

Table C.1. Posterior Distribution of the Estimated Parameters

	$\sigma = 1, \varphi = 0.5$	
	ψ_0	ψ_1
Posterior Mean	0.6140	0.4865
Standard Deviation	0.2728	0.2886
Posterior Mode	0.9700	0.0700
10th Percentile	0.1950	0.0921
90th Percentile	0.9429	0.8931

Figure C.1. Posterior Distribution of ψ_0 ($\sigma = 1$, $\varphi = 0.5$)

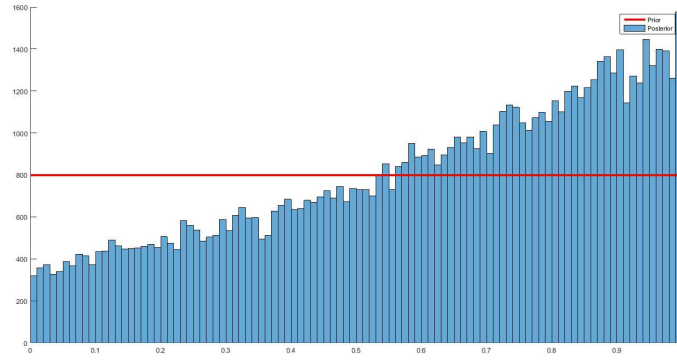
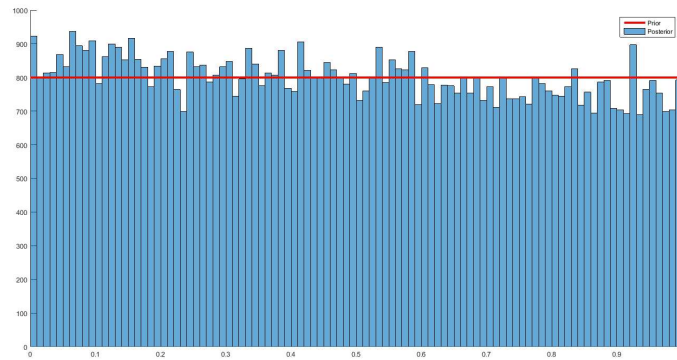


Figure C.2. Posterior Distribution of ψ_1 ($\sigma = 1$, $\varphi = 0.5$)



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